NAME:

Math 105D - Exam 2 - November 10, 2006

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (15 points) You have 54 ft^2 of material with which to build a box with a square base. One side (not the top or the bottom) of the box is to remain open. What is the maximum possible volume of the box? (Be sure to justify how you know this is a maximum.)

2. (10 points) The equation $x^3y + y^3 = 7 + x$ defines a graph. Using implicit differentiation, find $\frac{dy}{dx}$.

3. (10 points) Find the minimum and maximum values of the function $f(x) = \dfrac{x}{1+x^2}$ on the interval $[-3, 2]$.

4. (15 points) Suppose $g(x)$ is a function where $g(1) = 2, g'(1) = -4, g''(1) = 3$.

   (a) Find the equation of the tangent line to the graph of $y = g(x)$ at $x = 1$.

   (b) Suppose that $f(x) = e^x g(x)$. Determine the concavity of $f(x)$ at $x = 1$.

5. (8 points) Find all antiderivatives of $h(x) = x^2 + \sqrt{x} + \dfrac{1}{1 + 4x^2}$.

6. (16 points)

(a) Calculate $\lim_{x \to \infty} \dfrac{e^{2x} + x}{3x^2}$.

(b) Calculate $\lim_{x \to \pi} \dfrac{\sin(3x)}{x - \pi}$.

7. (10 points) For some function $g(x)$, suppose we know that there is a root at $x = a$ for some number a in the interval $(1, 4)$. Also suppose that $g(x)$ is increasing and concave down on the interval $[1, 4]$.

   (a) Draw the graph (labeled appropriately) of a function $g(x)$ that satisfies the above properties.

   (b) Using Newton's method, if our initial guess x_0 is in $[1, 4]$ and greater than a, will x_1 be less than a or greater than a, or can we not be sure? Explain. (Your graph above may be helpful.)

   (c) What if our initial guess x_0 is in [1,4] and less than a? Explain.

8. (16 points) Find derivatives of the following. DO NOT SIMPLIFY.

(a) If $f(x) = \left(\sin(\sqrt{x}) + \ln x + 5^{\pi}\right)^{10}$, find $f'(x)$.

(b) If $g(x) = \ln\left(\arctan\left(1/x\right)\right)$, find $g'(x)$.

1	2	3	4	5	6	7	8	TOTAL