Math 206 — Second Midterm

November 9, 2012

Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 7 pages including this cover AND IS DOUBLE SIDED. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	18	
2	18	
3	17	
4	15	
5	15	
6	17	
Total	100	

1. [18 points] In this problem, $f(x, y)$ is a function with continuous partial derivatives, and

$$\nabla f(20, -100) = (5, -2),$$

and $g(x, y) = f(xy^2, 2x^2y)$.

a. [6 points] What is $\nabla g(5, -2)$?.

b. [6 points] Let $n = (4, 2)$. What is the directional derivative $\frac{dg}{dn}(5, -2)$?

c. [6 points] Use the above to approximate $g(4, 0)$.

2. [18 points] In this problem, let $f(x, y) = 36x^3 + 108xy^2 - 507x - 360y$.

a. [6 points] The function $f(x, y)$ has a local maximum at what point?

b. [6 points] The function $f(x, y)$ has a local minimum at what point?

c. [6 points] The function has two saddle points at what points?

3. [17 points] The Apollo theatre has seating area that goes up and down, and is shaped in a semi-circle. The height of the floor is modeled by $f(x, y) = 5x^2 + 2y^2 - 10x$ feet above sea level, and the semi-circular shape is modeled by

$$x^2 + y^2 \leq 36, \quad \text{and} \quad x \geq 0.$$

a. [7 points] What are the critical points of the height of the theatre?

b. [5 points] What is the maximum height of the theatre?

c. [5 points] What is the minimum height of the theatre?

4. [15 points] Questions about div, grad, and curl.

a. [5 points] $F : \mathbb{R}^m \to \mathbb{R}^n$ is a function. If we can calculate the gradient of this function, what must be true about m and n?

b. [5 points] $G : \mathbb{R}^m \to \mathbb{R}^n$ is a function. If we can calculate the curl of this function, what must be true about m and n?

c. [5 points] $H : \mathbb{R}^m \to \mathbb{R}^n$ is a function. If we can calculate the the divergence of this function, what must be true about m and n?

5. [15 points] You like to buy chips and salsa at the store. Your utility function (your happiness) is $f(x, y) = xy^2 - x - y$ if you buy x units of chips and y units of salsa. The price per units of chips is $5 + 5/x$ if you buy x units, and the price per unit of salsa is always 10. You have \$ 20 to spend. How much chips and salsa should you buy to maximize your happiness?

6. [17 points] Consider the surface defined by

$$e^{x}\, y + 2\, e^{y}\, z - e^{z} = 0.$$

a. [5 points] What is $\dfrac{dz}{dx}$?

b. [5 points] What is $\dfrac{dx}{dy}$?

c. [5 points] What is $\dfrac{dy}{dz}$?

d. [2 points] What is $\dfrac{dz}{dx}\dfrac{dx}{dy}\dfrac{dy}{dz}$?