1. Suppose an economy with three sectors D, W and Z has consumption matrix $C = \begin{bmatrix} 0.01 & 0.02 & 0.05 \\ 0.10 & 0.06 & 0.07 \\ 0.15 & 0.03 & 0.04 \end{bmatrix}$.

Suppose $\mathbf{x}$, the vector which represents the total number of goods *actually produced* by the three sectors D, W and Z in supplying the final demands of an open sector is $\mathbf{x} = \begin{bmatrix} 3000 \\ 1000 \\ 8000 \end{bmatrix}$.

1A) Let $\mathbf{d}$ be the final demand vector of the open sector. What is the matrix equation involving $\mathbf{x}$, $\mathbf{d}$ and C?

1B) Explicitly find $\mathbf{d}$ for the vector $\mathbf{x}$ given above.

1C) Each unit produced by D requires how many units of Z's product?

1D) Of the total number of units actually produced by sector D, how many of them are consumed by sector Z?

2. Let $A = \begin{bmatrix} 11 & 2 & 0 & -6 \\ 0 & 7 & 0 & 0 \\ 33 & 0 & -4 & -33 \\ 4 & 2 & 0 & 1 \end{bmatrix}$.

2A) Find the characteristic polynomial of A. Show all your work. Be smart and take advantage of the many zeros in this matrix.

2B) Find the eigenvalues of A, and their multiplicities.

3. Suppose the determinant of some 4x4 matrix M is 3. Next to each of the following matrices, write its determinant.

M^5 $\quad$ $5M$ $\quad$ $M + 2M + 7M$ $\quad$ $-M$

M^{-1} $\quad$ M^T $\quad$ $(M^{-1})^T$ $\quad$ $((5M)^{-1})^T$

4. Let $M = \begin{bmatrix} 9 & -8 & 20 \\ 8 & -11 & 40 \\ 2 & -4 & 15 \end{bmatrix}$.

4A) It's a fact that $\lambda = 5$ is an eigenvalue of M. Find a basis for the corresponding eigenspace.

4B) Let $\mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$. Find $M\mathbf{v}$. Then determine if the result says that $\mathbf{v}$ is an eigenvector of M. If so, what's the corresponding eigenvalue?

4C) The matrix M is diagonalizable. Find P and D for which $M = PDP^{-1}$ where D is a diagonal matrix consisting of the eigenvalues of M, P is invertible, and the column vectors in P are eigenvectors corresponding to the eigenvalues in D.

$P =$ $D =$

4D) Use your calculator to find P^{-1}. What is it?

5. Suppose that $A = \begin{bmatrix} p & 3 & 5 \\ -7 & x & 0 \\ 4 & 0 & 1 \end{bmatrix}$.

For each matrix below, determine if that matrix is row equivalent to A. If so, find elementary matrices that represent the row operations done to A that turn it into the given matrix. List your matrices in the order you need to multiply them by in order for their product to turn A into the given matrix.

5a) $A_1 = \begin{bmatrix} 3p & 9 & 15 \\ 1 & x & 2 \\ 4 & 0 & 1 \end{bmatrix}$

5b) $A_2 = \begin{bmatrix} -7 & x & 0 \\ 5p & 15 & 25 \\ 1 & 0 & 1/4 \end{bmatrix}$.

5c) Now, suppose that det(A)=24. What are the determinants of A_1 and A_2?

$\det(A_1) = \boxed{}$ $\det(A_2) = \boxed{}$

6. Let H be a subspace of $\mathbb{R}^k$ for some k.

6a) What does it mean to say (ie, give the definition) that a set $\mathcal{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_p\}$ of vectors in H is a *basis* of H?

6b. What is the *dimension* of H? (again, give the *definition*).

7. Let $K = \begin{bmatrix} 2 & 3 & -6 & -7 \\ 1 & 2 & -1 & -1 \\ 1 & 4 & 7 & 9 \\ 3 & 8 & 5 & 7 \end{bmatrix}$ and let $\mathbf{s} = \begin{bmatrix} 11 \\ 5 \\ 3 \\ 13 \end{bmatrix}$. Label the columns of K as $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$, and $\mathbf{k}_4$.

You do **not** need to check these three facts:

(1) The vector $\mathbf{s}$ is in $\mathrm{Col}(K)$.

(2) The set $\mathcal{B} = \{\mathbf{k}_1,\ \mathbf{k}_2\}$ is a basis for $\mathrm{Col}(K)$.

(3) The set $\mathcal{D} = \{\mathbf{k}_3,\ \mathbf{k}_4\}$ is a basis for $\mathrm{Col}(K)$.

7a) Find $[\mathbf{s}]_{\mathcal{B}}$. (Show any relevant work in all parts of this problem).

7b) Find $[\mathbf{s}]_{\mathcal{D}}$.

7c) Find $[\mathbf{k}_1]_{\mathcal{B}}$.

7d) Find the dimension of $\mathrm{Col}(K)$.

7e) Find the dimension of $\mathrm{Nul}(K)$.