

**Math 105: Review for Exam II**

1. Find  $dy/dx$  for each of the following.

(a)  $y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + \arctan 2$

(b)  $y = \sqrt{x} \cdot \arctan(5x)$

(c)  $y = \ln(\tan(2^{\cos(x^2)}))$

(d)  $y = \sin^5\left(\frac{x + e^\pi}{\ln 4 + \arcsin 6x}\right)$

(e)  $y = (x^2 + 1)^{\sin x}$

[The 1:10 section may omit this problem.]

2. Consider the curve defined by  $x^3 + y^3 = \frac{9}{2}xy$  (known as the Folium of Descartes).

(a) Find  $dy/dx$ .

(b) Find the equation of the tangent line at the point (1,2).

3. Evaluate the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

(b)  $\lim_{x \rightarrow \infty} \frac{e^x}{\ln x}$

(c)  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3x}$

(d)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{7 - 7x}$

(e)  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{5x^2}$

4. Suppose the domain of  $f(x)$  is all reals and that  $f$  has an inverse function  $f^{-1}(x)$ . Further, suppose that  $f(2) = 5$  and  $f'(2) = e$ . Finally, let  $h(x) = 1/f(x)$ .

(a) What point must be on the graph of  $f^{-1}(x)$ ?

(b) What point must be on the graph of  $h(x)$ ?

(c) Give an example of a point that cannot be on the graph of  $f(x)$ . Do not choose a point with  $x$ -value of 2.

(d) What is the value of the derivative of  $h(x)$  at  $x = 2$ ?

(e) What is the value of the derivative of  $f^{-1}(x)$  at  $x = 5$ ?

5. Suppose that  $y = f(t)$  is a solution to the differential equation  $y' = \frac{1}{\pi} \arcsin t + y^2$  and that  $f\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$ . Find the equation of the tangent line to  $f$  at  $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ .

6. Simplify  $\tan(\arccos x)$  as much as possible.

7. Consider the function  $f(x) = x^4 e^x$  with domain all real numbers.

(a) Find the  $x$ -value(s) of all roots ( $x$ -intercepts) of  $f$ .

(b) Find the  $x$ - and  $y$ -value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the  $x$ - and  $y$ -value(s) of all global extrema and identify each as a global max or global min.

(d) Find the  $x$ -value(s) of all inflection points.

(e) Sketch  $f$ .

8. How would your answers to the previous question change if the domain of  $f$  were  $[-10, 10]$ ?

9. Circle *always*, *sometimes*, or *never* to make each statement below correct.

(a) If  $f'(1) = 0$  then  $f$  *always/sometimes/never* has a critical point at  $x = 1$ .

(b) If  $f'(2) = 0$  then  $f$  *always/sometimes/never* has a local maximum or local minimum at  $x = 2$ .

(c) If  $x = 3$  is a critical point of  $f$ , then  $f'(3)$  is *always/sometimes/never* 0.

(d) If  $f''(4) = 0$ , then  $f$  *always/sometimes/never* has an inflection point at  $x = 4$ .

(e) If  $f$  has a global maximum at  $x = 5$ , then  $f'(5)$  is *always/sometimes/never* 0.

(f) If  $f'(6) = 0$  and  $f''(6) = -2$ , then  $f$  *always/sometimes/never* has a local maximum at  $x = 6$ .

(g) If  $f'(7) = 0$  and  $f''(7) = 0$ , then  $f$  *always/sometimes/never* has a local extremum at  $x = 7$ .

10. The rate of change of a population  $P(t)$  of eels is proportional to the size of the population. When the population is 40000, it is growing at a rate of 400 eels per year. At time  $t = 0$ , the population is 10000.

(a) Write a differential equation whose solution is  $P(t)$ .

(b) Solve your differential equation.

(c) When will the population reach 60000?

11. Use Newton's Method with an initial guess of  $x_0 = -1$  to find the next three approximations to a solution of  $x^3 + x + 1 = 0$ . Then test your final approximation to see if it appears to be close to a root.

12. You are designing an  $18 \text{ ft}^3$  box that will have a square bottom and no top. The material for the bottom costs 40 cents per square foot and the material for the sides costs 30 cents per square foot. What dimensions give the least total cost? Be sure to show how you know you have found the minimum.