1. (a) For any point on \( x^2 + y^2 = 4 \), no ball around it is contained in \( A \).
   So \( A \) is not open.
   For any point on \( x^2 + y^2 = 9 \), no ball around it is contained in the complement of \( A \).
   So complement is not open. So \( A \) is not closed.

   Boundary of \( A = \{(x, y) \mid x^2 + y^2 = 4 \text{ or } x^2 + y^2 = 9\} \)

   Complement of \( A = \{(x, y) \mid x^2 + y^2 < 4 \text{ or } x^2 + y^2 \geq 9\} \)

1. (b) \( B \) consists of all the points on the plane and above the plane.

   For any point in complement of \( B \), there is a ball around it that is contained in the complement. So the complement is open.
   \( B \) is closed.

   Boundary of \( B = \{(x, y, z) \mid y \leq 0\} \)

   Complement of \( B = \{(x, y, z) \mid y < 0\} \)

1. (c) \( C \) consists of the points on the sphere and outside the sphere.

   For any point in \( C \), there is a ball around it that is contained in \( C \). So \( C \) is open.

   Boundary of \( C = \{(x, y, z) \mid \| (x, y, z) - (1, 1, 1) \| = 2\} \)

   Complement of \( C = \{(x, y, z) \mid \| (x, y, z) - (1, 1, 1) \| \geq 2\} \)

2) Circle of radius 2, centered at \((1, 2)\).
   (Inside of circle is the ball set \( B_2(1,2) \)).
3) \[ \lim_{{(x,y) \to (0,0)} \atop y=0} \frac{x^2 y^2}{x^4+y^4} = 0 \quad \lim_{{x \to 0} \atop (x,y) \to (0,0)} \frac{x^2 y^2}{x^4+y^4} = 0 \]
\[ \lim_{{(x,y) \to (0,0)} \atop y=x} \frac{x^2 y^2}{x^4+y^4} = \lim_{{x \to 0} \atop (x,y) \to (0,0)} \frac{x^4}{x^4+x^4} = \frac{1}{2}. \] So limit does not exist.

4) (a) (0,0) — only point of discontinuity since \( f(0,0) \) is not defined.
\[ \lim_{{(x,y) \to (0,0)} \atop (x,y) \to (0,0)} f(x,y) = \lim_{{(x,y) \to (0,0)} \atop (x,y) \to (0,0)} \frac{x^2 (x+1)+y^2 (x+1)}{x^2+y^2} = \lim_{{(x,y) \to (0,0)} \atop (x,y) \to (0,0)} (x+1) = 1 \]
Since the limit exists, the discontinuity is removable.

4) (b) \( g(0,0,0) \) is not defined.
\( (0,0,0) \): only point of discontinuity
\[ \lim_{{(x,y,z) \to (0,0,0)} \atop y=z=0} g(x,y,z) = \lim_{{x \to 0} \atop x \to 0} \frac{3x^4}{x^4} = 3 \] \( \lim_{{(x,y,z) \to (0,0,0)} \atop x \to 0} g(x,y,z) \) does not exist.
Discontinuity is not removable.

4) (c) \( f(0,0) \) not defined. (0,0): only point of discontinuity.
Let \( u = x^2+y^2 \).
\[ \lim_{{(x,y) \to (0,0)} \atop u \to 0^+} f(x,y) = \lim_{{u \to 0^+} \atop u \to 0^+} \frac{\ln (1-u)+u}{u} = 0 \] (L'Hôpital's rule)
So discontinuity is removable.
\( f(x,y) \) is also not defined when \( x^2+y^2 \geq 1 \) since \( \ln (1-x^2-y^2) \) is not defined at those points.
\[ f(x,y) \] is defined only inside the circle, except at \((0,0)\).
We ignore the points outside the circle.
(Check at Defn. 3.2.2 on page 162 of text)
Other points of discontinuity: \( \{(x,y) \mid x^2+y^2=1\} \).
Let \( u = x^2 + y^2 \); \( a = (a_1, a_2) \): any point on circle
\[
\lim_{(x,y) \to (a_1, a_2)} f(x,y) = \lim_{u \to 1^-} \frac{\ln(1 - u) + u}{u} = -\infty \quad \text{as } u \to 1^-.
\]
So discontinuities are not removable.

4(d) Please ignore this problem.

5) For \( (x,y) \neq (0,0) \),
\[
f_x = \frac{8xy^3}{(x^2 + y^2)^2}, \quad f_y = \frac{3x^4 - 6x^2y^2 - y^4}{(x^2 + y^2)^2}.
\]
\[
\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(0+h, 0) - f(0,0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0\;
\]
\[
\frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f(0, 0+h) - f(0,0)}{h} = \lim_{h \to 0} \frac{-h - 0}{h} = -1
\]

6(a) \( P: \mathbb{R}^3 \to \mathbb{R} \) \( P(x,y,z) = \frac{6x^2z}{y} \).
\[
\frac{\partial P}{\partial x} = \frac{12xz}{y}, \quad \frac{\partial P}{\partial y} = \frac{-6x^2z}{y^2}, \quad \frac{\partial P}{\partial z} = \frac{6x^2}{y}.
\]
\[
\frac{dP}{dt} = \frac{dP}{dt}(f(t)) = \frac{d}{dt} \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial y} \\ \frac{\partial P}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} \frac{dP}{dt}(f(t))
\]

So
\[
\frac{dp}{dt} = \frac{dx}{dt} \frac{\partial P}{\partial x} + \frac{dy}{dt} \frac{\partial P}{\partial y} + \frac{dz}{dt} \frac{\partial P}{\partial z}
\]

At \( t = \frac{\pi}{4} \),
\[
\frac{dp}{dt} = \frac{12xz}{y} \left( -2 \sin t \right) - \frac{6x^2z}{y^2} \left( 2 \cos t \right) + \frac{6x^2}{y} (3)
\]
\[\frac{dp}{dt} \bigg|_{t = \frac{\pi}{4}} = \frac{36 - 27\pi}{\sqrt{2}} = P' \left( \frac{\pi}{4} \right)
\]

6(b) \( P \left( \frac{\pi}{4} + 0.01 \right) \approx P \left( \frac{\pi}{4} \right) + P' \left( \frac{\pi}{4} \right) \left( 0.01 - \frac{\pi}{4} \right) \approx \frac{9\pi}{\sqrt{2}} - 0.34523 \approx 19.6477 \).
7. (a) \[ J_f(x,y) = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix} \quad J_f(1,0) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \]

\[ D_f(a) (x,y) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} \]

1e \[ D_f(a)(x,y) = (2x,2y) \]

7. (b) \( D_f(a) \) scales each vector by 2 in the x-direction and in the y-direction.

7. (c) \[ f(0.99,0.01) \approx f(0,1,0) + D_f(a)(0.99,0.01) - (1,0) \]

\[ = (1,0) + D_f(a)(-0.01,0.01) \]

\[ = (1,0) + (-0.02,0.02) \]

\[ = (0.98, 0.02) \]

8. (a) \[ dT = 16 \, dx - 15 \, dy \quad (T_x(2,1) = 16, \, T_y(2,1) = -15) \]

8. (b) \[ T(2.04,0.97) = T(2,1) + dT = T(2,1) + 16(2.04-2) - 15(0.97-1) \]

\[ = T(2,1) + 16(0.04) - 15(-0.03) = 136.09^\circ C \]

9) \[ T_x(x,y,t) = -e^{-kt} \sin x, \quad T_{xx}(x,y,t) = -e^{-kt} \cos x \]

\[ T_y(x,y,t) = -e^{-kt} \sin y, \quad T_{yy}(x,y,t) = -e^{-kt} \cos y \]

\[ T_t(x,y,t) = -ke^{-kt} (\cos x + \cos y) = k(T_{xx}(x,y,t) + T_{yy}(x,y,t)) \]

10. (a) \( f(1,2) = 7 \) means temperature is \( 7^\circ C \) at the point (1,2).

\( f_x(1,2) = -3 \) means temperature decreases at a rate of \( 3^\circ C/cm \) in the x-direction at the point (1,2).

10. (b) \( \nabla f(1,2) = -(-3,4) = (3,-4) = 3\mathbf{i} - 4\mathbf{j} \)

10. (c) \( \nabla f = (3,4) \) is perpendicular to the level curve.
11. \( \nabla f \| = 5 \). At the point \((1,2)\), the maximum rate of change of temperature \((5^\circ C/cm)\) occurs in the direction \(-3\mathbf{i} + 4\mathbf{j}\).

(Another possible answer: \(-\nabla f\))

10 (d) \( \nabla f(1,2) \cdot (\mathbf{c}(x,y)-(1,2)) = 0 \); \(-3(x-1)+4(y-2)=0\).

11) \( f(t) = (\cos t, \sin t, t) \); \(0 \leq t \leq 4\pi\).

\[ \int_{C} \mathbf{F} \cdot d\mathbf{s} = \int_{0}^{4\pi} (-\sin t, \cos t, 5) \cdot (-\sin t, \cos t, 1) \, dt = \int_{0}^{4\pi} 6 \, dt = 24\pi. \]

12) Problem 11 on page 160

\[ f(x,y) = c \]

\[ \sqrt{4-x^2-y^2} = c \]

\[ 4-x^2-y^2 = c^2 \]

\[ x^2+y^2 = 4-c^2. \]

\[ f(1,1) = \sqrt{2}. \]

So \((1,1)\) is a point on the level curve for \(\sqrt{2}\).

13) Problem 23 on page 161.

Answer at the back of the book.

Orient any of the two paths drawn towards the origin.

Then line integral will be negative.

Remaining problems: odd numbered problems from the textbook.