1. Let $A = \begin{bmatrix} 2 & 5 & b \\ a & 6 & 1 \\ 9 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 & 2 \\ c & 5 & d \\ 1 & 0 & 9 \end{bmatrix}$ and suppose $AB = \begin{bmatrix} 57 & 25 & 82 \\ 64 & 30 & 33 \\ 49 & 0 & 54 \end{bmatrix}$.

1a. What equations involving one or more of a, b, c, and d need to be solved in order to find a, b, c, and d? Are they all linear?

1b. Solve the system involving the linear equations in (1a). Use augmented matrices where appropriate; show your augmented matrix/matrices.

2. A model of an economy shows four sectors M, A, T, and H. Output from each sector is distributed as follows: M requires none of H's output, and the other three sectors consume equal amounts of the output of H. None of M's output is used by H and M, A, and T receive equal amounts of M's output. M, T and H use 10, 40 and 30% of A's output respectively; A uses the rest for itself. Finally, T uses none of its own output, half of which is consumed by M and the remainder split between A and H.

2a. What is the exchange table for this model?

2b. Find a set of equilibrium prices for P_M, P_A, P_T, and P_H so that each sector's income matches its expenses, assuming that P_H is 81.

3. Let $\overline{S}$ be the familiar vector space of all sequences $\mathbf{s} = (s_1,\ s_2,\ s_3,\ s_4,\ \dots)$ of real numbers such that all but finitely many of the entries of $\mathbf{s}$ are 0. Suppose $T : \overline{S} \to \overline{S}$ is defined by

$$T(\mathbf{s}) = T((s_1,\ s_2,\ s_3,\ s_4,\ \dots)) = (s_1 - s_2,\ s_2 - s_1,\ s_3 - s_4,\ s_4 - s_3, \dots).$$

3a. Find $T((2, 4, 3, 0, 8, 1, 0, 0, 0, 0, 0, 0 \dots))$.

3b. Show that T is a linear transformation.

3c. What form does $\mathbf{s} = (s_1,\ s_2,\ s_3,\ s_4,\ \dots)$ have to have in order for $\mathbf{s}$ to be in $\mathrm{Ker}(T)$?

3d. What form does $\mathbf{b} = (b_1,\ b_2,\ b_3,\ b_4,\ \dots)$ have to have in order for $\mathbf{b}$ to be in $\mathrm{Im}(T)$?

3e. Is T one-to-one? *Explain!*

3f. Is T onto $\overline{S}$? *Explain!*

4. Let $D = \begin{bmatrix} 1 & 1 & 3 & 3 & 8 \\ 1 & 1 & 4 & 5 & 10 \\ 3 & 3 & 11 & 14 & 29 \\ 2 & 2 & 7 & 6 & 16 \end{bmatrix}$ and label its five columns $\mathbf{c}_1$, $\mathbf{c}_2$, $\ldots \mathbf{c}_5$.

4a. Find a basis for Col(D).

4b. Find a basis for Col(RREF(D)).

4c. Find a basis for Nul(D).

4d. Find a basis for Nul(RREF(D)).

4e. What is the rank of D?

4f. Is $\mathbf{c}_3$ a linear combination of the other columns? Explain.

5. All questions on this page are "short answer". **** **No** proofs, **no** explanations required! ****

Note that "DNE" (for "Does Not Exist") is a possible answer.

5a. Dim($\mathbf{R}^4$) is?

5b. A basis for $\mathbb{P}_3$ is?

5c. Dim(Nul(I_4)) is?

5d. The inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is?

5e. A finite set that spans($\overline{S}$) is?

5f. A finite set that spans $\{\mathbf{0}\}$ is?

Let H be the subset of all functions in $\mathbb{F}$ whose graph never goes below the x-axis.

5g. Is the 0-vector of $\mathbb{F}$ a member of H? (Y/N)

5h. Is H closed under vector addition? (Y/N)

5i. Is H closed under scalar multiplication? (Y/N)

5j. $\mathbb{P}_3$ is a subspace of $\mathbb{P}_4$ of dimension 4. (T/F)

5k. $\mathbf{R}^3$ is a subspace of $\mathbf{R}^4$ of dimension 3. (T/F)