NAME: SOLUTIONS

Math 105 - Quiz 11 - November 7, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (10 pts.) What is the minimum value of the expression $x^3 + y$ if $x + y = 4$ and $x \geq 0$? Justify your answer. (Your final answer should be exact, rather than a decimal approximation. For instance, if the final answer is π, then writing 3.14 is not exact.)

Objective fn $= x^3 + y$.

Constraint: $x + y = 4 \rightarrow y = 4 - x$.

Objective $= x^3 + y = x^3 + 4 - x$. Find critical pts.

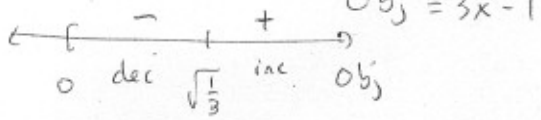
$\text{Obj}' = 3x^2 - 1 \overset{\text{set}}{=} 0$.

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$.

But $x \geq 0$, so $x = \sqrt{\frac{1}{3}}$.



The objective fn $x^3 + y$ has a global minimum value at $x = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$.

2. (10 pts.) Evaluate $\lim_{x \to 0} \frac{-1 + \cos x}{x^2}$. If you use l'Hôpital's rule, mention it and justify your usage.

$\lim_{x \to 0} \frac{-1 + \cos x}{x^2} = \frac{-1 + 1}{0} = \frac{"0"}{0}$. Indet form, use l'H.

$\overset{H}{=} \lim_{x \to 0} \frac{-\sin x}{2x} = \frac{"0"}{0}$. Indet form, use l'H.

$\overset{H}{=} \lim_{x \to 0} \frac{-\cos x}{2} = \boxed{\frac{-1}{2}}$