Name:_____

Exam 2- Take-Home Portion

Show all your work to receive full credit for a problem.

Attach this sheet to the solutions you hand in. Even if you attempt the problems in any order, write the solutions in the chronological order.

Please sign and date the following statement:

I declare that the work I am submitting is entirely my own and that I did not confer with anyone (except maybe the instructor) in completing this exam. Further, I declare that I did not use any sources other than my class notes and the course textbook.

Signature: _____

Date: _____

7. Let C be the curve parametrized by $\mathbf{f}(t) = (t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3})$ where $-2 \leq t \leq 2$.

   (a) (3 pts) Show C is smooth and simple.

   (b) (6 pts) Find the length of C.

8. (15 pts) Let $\mathbf{f}(t) = (\cos t, \sin t)$, where $0 \leq t \leq \pi$ and $\mathbf{g}(t) = (-t, \sqrt{1-t^2})$ where $-1 \leq t \leq 1$, and let $\mathbf{F}(x, y) = (3x - y, x)$.

   (a) Show that $\mathbf{f}$ and $\mathbf{g}$ both parametrize the same curve C (Hint: First establish what C should be.) What is the orientation of each path?

   (b) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{x}$ along both of the parametrizations and verify that your answers coincide. Explain why this makes sense (without writing a proof).

   (c) Now compute the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{x}$ along the path γ parametrized by $\mathbf{h}(t) = (-t, t^2 - 1)$ where $-1 \leq t \leq 1$. Notice that the endpoints of γ and C are the same. Is your answer the same as what you got in part (b)?

   (d) Let $\mathbf{G}(x, y) = (x + 2, 2y + 3)$. Compute the line integrals $\int_C \mathbf{G} \cdot d\mathbf{x}$ and $\int_\gamma \mathbf{G} \cdot d\mathbf{x}$ and compare the answers.

   (e) Write a statement that summarizes your findings in this problem.