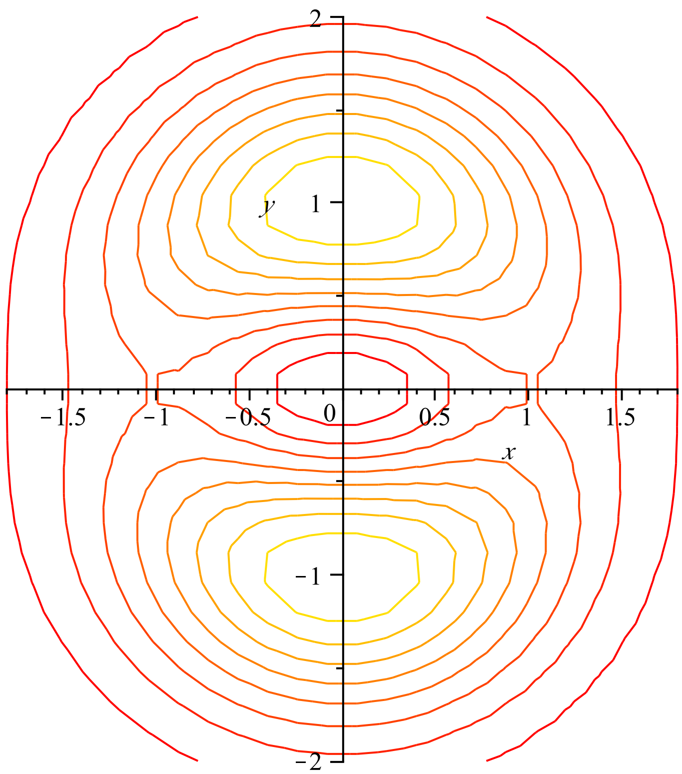
Name:____

Exam 2- In-Class Portion

Show all your work to receive full credit for a problem.

1. Let f be a two-variable, real valued function with level curves as pictured below.

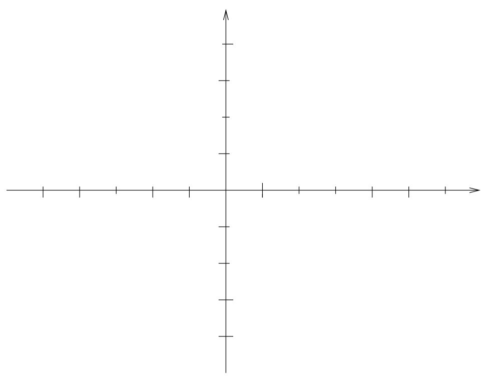


(a) (3 pts) At the point (1/2,1/2), draw the vector $\mathbf{v} = \frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$. Is the directional derivative in the direction of $\mathbf{v}$ going to be positive, negative, or zero? Explain your reasoning.

(b) (3 pts) At the same point, draw the approximate direction of the gradient, and briefly explain your reasoning.

2. Let $S = \{(x, y) | -1 \leq x \leq 1, x^2 \leq y \leq 1\}$.

(a) (3 pts) Draw S in the coordinate axes below.



(b) (3 pts) Is S open, closed or neither?

(c) (3 pts) Is S bounded?

(d) (3 pts) Let $f(x, y) = x^4 y^5 \sin x \cos y$. Does f attain a maximum value on S? Does f attain a minimum value on S? For both questions, explain why or why not.

3. (6 pts) Let $u(x,t) = e^{3t}\sin x$. Show that u satisfies the following partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{3}\frac{\partial u}{\partial t} = 0$$

4. (6 pts) Suppose $f(x,y)$ is a differentiable function with $\frac{\partial f}{\partial x}(4,1) = 2$ and $\frac{\partial f}{\partial y}(4,1) = -1$. Find the equation of the tangent line to the level curve of f which goes through the point (4,1).

5. (a) (6 pts) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable at (0,0), satisfies $f(0,0) = 4$ and has Jacobian matrix

$$Jf(0,0) = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

Explain how you would use these values to find an approximation for $f(x,y)$ where (x,y) is very close to the origin.

(b) (6 pts) Now suppose $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^2$ is differentiable at $(1,0,-1)$, satisfies $\mathbf{g}(1,0,-1) = (0,0)$ and has Jacobian matrix

$$J\mathbf{g}(1,0,-1) = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

Explain how you can use these values and the values in part (a) to find an approximation for $(f \circ \mathbf{g})(x,y,z)$ where (x,y,z) is very close to $(1,0,-1)$.

6. Let f be a real-valued, two variable function which has critical points

$$(1,0),(-1,0),(1,4),(-1,4)$$

and at these points, the Hessians are

$$Hf(1,0) = \begin{pmatrix} 6 & 0 \\ 0 & -12 \end{pmatrix} \qquad Hf(-1,0) = \begin{pmatrix} -6 & 0 \\ 0 & -12 \end{pmatrix}$$

$$Hf(1,4) = \begin{pmatrix} 6 & 0 \\ 0 & 12 \end{pmatrix} \qquad Hf(-1,4) = \begin{pmatrix} -6 & 0 \\ 0 & 12 \end{pmatrix}$$

(a) (6 pts) Fill in the following equalities, and briefly explain your reasoning.

$\frac{\partial^2 f}{\partial x^2}(-1,0) =$

$\frac{\partial^2 f}{\partial y \partial x}(1,4) =$

$\nabla f(-1,4) =$

(b) (6 pts) Classify the critical points as local maxima, local minima, saddle points, or none of these.