NAME:

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers.

Advice: DON’T spend too much time on a single problem.

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1. (7 pts) (i) Consider the function

\[ f(x, y) = \begin{cases} 
\frac{3xy}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0); \\
0, & \text{otherwise.}
\end{cases} \]

Determine whether \( f(x, y) \) is continuous at \((0, 0)\). Justify your answer. [Hint: try approaching \((0, 0)\) from different directions]

(8 pts.) (ii) Consider the vector field \( F(x, y, z) = (2xz, -xy, -z) \). Find \( \text{div} \ F \) and \( \text{curl} \ F \).
2. Let \( f : \mathbb{R}^2 \to \mathbb{R}^3 \) be defined by \( f(x, y) = (\cos(\pi x), \sin(\pi x), e^{x+y}) \). Let \( g : \mathbb{R}^3 \to \mathbb{R}^2 \) be defined by \( g(x, y, z) = (x + y, \ln z) \).

(4 pts) (i) Find the Jacobian matrix \( J(f) \).

(4 pts) (ii) Find the Jacobian matrix \( J(g) \).

(6 pts) Find \( J(g \circ f)(0, 1) \).

(6 pts) Find \( J(f \circ g)(1, 0, e) \).
3. Let \( f(x, y) = x^2 e^{-2y} \).

(i) Find the directional derivative \( D_u f(1, 0) \) of \( f \) at the point \((1, 0)\) in the direction of \( u = i + j \).

(ii) Find an equation for the line tangent to the level curve \( f(x, y) = 1 \) at the point \((1, 0)\).
4. Consider the function \( f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 \).

(4 pts) (i) Find all the critical points of \( f \).

(8 pts) (ii) For each of the critical point(s) \( a \) found in part (i), find the corresponding Hessian matrix \( Hf(a) \).

(8 pts) (iii) Use the second derivative test to classify each of the critical point(s) in part (i), i.e., determine whether the critical point is a local max, local min, or saddle point.
5. Let $C$ be a smooth path in $\mathbb{R}^3$ given by the parametrization

$$x(t) = (2t, t, 2 - 2t)$$

for $0 \leq t \leq 1$.

(7 pts) (i) Find the length of the path $C$.

(8 pts.) (ii) Suppose a vertical wall is to be built on top of the path $C$ whose height is given by

$$f(x, y, z) = xy + y + z.$$ 

Find the surface area of this wall.
6. Let $C$ be the arc of the unit circle from $(1,0)$ to $(0,1)$ and $F(x,y) = (x, x^2 + y^2)$.

(6 pts.) Write a parametrization $\mathbf{x}(t)$ for the curve $C$. Be sure to state the range of the parameter.

(9 pts.) Find the work done by the force $F$ over the curve $C$. That is, find $\int_C F(\mathbf{x}) \cdot d\mathbf{x}$. 