MATH206A MULTIVARIABLE CALCULUS - PROF. P. WONG

EXAM II - NOVEMBER 6, 2007

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	15	
2.	20	
3.	15	
4.	20	
5.	15	
6.	15	
Total	100	

1.(7 pts) (i) Consider the function

$$f(x,y) = \begin{cases} \frac{3xy}{x^4+y^2}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{otherwise.} \end{cases}$$

Determine whether $f(x,y)$ is a continuous at $(0,0)$. Justify your answer. [Hint: try approaching $(0,0)$ from different directions]

(8 pts.)(ii) Consider the vector field $F(x,y,z) = (2xz, -xy, -z)$. Find div F and curl F.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(x, y) = (\cos(\pi x), \sin(\pi x), e^{x+y})$. Let $g : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $g(x, y, z) = (x + y, \ln z)$.

(4 pts) (i) Find the Jacobian matrix $J(f)$.

(4 pts) (ii) Find the Jacobian matrix $J(g)$.

(6 pts) Find $J(g \circ f)(0, 1)$.

(6 pts) Find $J(f \circ g)(1, 0, e)$.

3. Let $f(x, y) = x^2 e^{-2y}$.

(7 pts) (i) Find the directional derivative $D_{\mathbf{u}} f(1, 0)$ of f at the point $(1, 0)$ in the direction of $\mathbf{u} = \mathbf{i} + \mathbf{j}$.

(8 pts) (ii) Find an equation for the line tangent to the level curve $f(x, y) = 1$ at the point $(1, 0)$.

4. Consider the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$.

(4 pts) (i) Find all the critical points of f.

(8 pts) (ii) For each of the critical point(s) $\mathbf{a}$ found in part (i), find the corresponding Hessian matrix $Hf(\mathbf{a})$.

(8 pts) (iii) Use the second derivative test to classify each of the critical point(s) in part (i), i.e., determine whether the critical point is a local max, local min, or saddle point.

5. Let C be a smooth path in $\mathbb{R}^3$ given by the parametrization

$$\mathbf{x}(t) = (2t, t, 2 - 2t)$$

for $0 \leq t \leq 1$.

(7 pts) (i) Find the length of the path C.

(8 pts.) (ii) Suppose a vertical wall is to be built on top of the path C whose height is given by

$$f(x, y, z) = xy + y + z.$$

Find the surface area of this wall.

6. Let C be the arc of the unit circle from $(1,0)$ to $(0,1)$ and $F(x,y) = (x, x^2 + y^2)$.

(6 pts.) Write a parametrization $\mathbf{x}(t)$ for the curve C. Be sure to state the range of the parameter.

(9 pts.) Find the work done by the force F over the curve C. That is, find $\int_C F(\mathbf{x}) \cdot d\mathbf{x}$.