NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

*Advice*: DON’T spend too much time on a single problem.

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1. Determine whether each of the following improper integrals converges or diverges. Justify your answers.

(10 pts.)(a) \[ \int_{1}^{\infty} \frac{1}{(1 + x^2)^3} \, dx \]
[Hint: compare this integral with another improper integral]

(10 pts.)(b) \[ \int_{0}^{1} \frac{1}{3x - 2} \, dx \]
[Hint: this is an improper integral]
2. (10 pts.) (a) Find the indefinite integral
\[ \int x e^{2x} \, dx. \]

(10 pts.) (b) Find the exact value, using the Fundamental Theorem of Calculus, of
\[ \int_2^3 \frac{3x}{(x - 1)(2x + 1)} \, dx. \]
3. Suppose a function $f$ satisfies
\[ f(1) = 2, \quad f'(1) = 3, \quad f''(1) = -2, \quad f'''(1) = 3, \quad f^{(4)}(1) = -5. \]

(a) Write down the fourth-degree Taylor polynomial $P_4(x; 1)$ for $f$ near $x = 1$.

(b) Suppose it is known that for $0 \leq x \leq 2.5$, $|f^{(5)}(x)| \leq 0.5$. What is the maximum possible error committed by using $P_4(x; 1)$ to estimate $f(x)$ for $0 \leq x \leq 2.5$?

(c) If $g(x) = f(x + 1)$, find the third-degree Maclaurin polynomial for $g$. [Hint: Use part (a).]
4. (10 pts.) (a) Let $f(x) = e^{2x+1}$. Find the fourth-degree Maclaurin polynomial $M_4(x)$ for $f$.

(10 pts.) (b) Given an income stream at the rate of $p(t) = 50,000e^{0.04t}$ at time $t$ years for the next 10 years, find the present value of this income stream if the annual interest rate is $r = 0.07$. 
5. (10 pts.) Find the indefinite integral
\[ \int \tan^3(\theta) \sec(\theta) \, d\theta. \]

(10 pts.) (b) Find the indefinite integral
\[ \int \frac{1}{\sqrt{10 + 2x + x^2}} \, dx. \]