MATH206A MULTIVARIABLE CALCULUS - PROF. P. WONG

EXAM II - NOVEMBER 3, 2006

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $F(x, y, z) = (yz, xz, xy)$.

(5 pts) (i) Find div F.

(5 pts) (ii) Find curl F

(5 pts) (iii) What is the Jacobian matrix $DF(1, 1, 1)$ of F at $(1, 1, 1)$?

(5 pts) (iv) Find an approximation of $F(0.9, 1.1, 1.1)$.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (2xy, 3x - y + 5)$.
(10 pts) (i) Suppose that $g : \mathbb{R}^3 \to \mathbb{R}^2$ is a differentiable function such that $g(1, -1, 3) = (2, 5)$ and $Dg(1, -1, 3) = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{bmatrix}$.

What is $D(f \circ g)(1, -1, 3)$?

(10 pts) (ii) Suppose that $h : \mathbb{R}^2 \to \mathbb{R}^2$ is another differentiable function such that $Dh(-2, 1) = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$. What is $D(h \circ f)(-1, 1)$?

3. Consider the function $f(x, y, z) = x^3y - yz^2 + z^5$.

(8 pts) (i) Find the directional derivative $D_{\mathbf{u}}f(1, 1, 0)$ of f at the point $(1, 1, 0)$ in the direction of $\mathbf{u} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(12 pts) (ii) Find an equation for the plane tangent to the level surface $f(x, y, z) = 9$ at the point $(3, -1, 2)$.

4. Consider the function $f(x, y) = 4y - y^3 - x^2$.

(7 pts) (i) Find all the critical points of f.

(6 pts) (ii) For each of the critical point(s) $\mathbf{a}$ found in part (i), find the corresponding Hessian matrix $Hf(\mathbf{a})$.

(7 pts) (iii) Use the second derivative test to classify each of the critical point(s) in part (i), i.e., determine whether the critical point is a local max, local min, or saddle point.

5. Let C be a smooth path in $\mathbb{R}^3$ given by the parametrization

$$\mathbf{x}(t) = (\ln t, t^2/2, \sqrt{2}t)$$

for $1 \leq t \leq 4$.

(6 pts) (i) Find the length of the path C. [Do your algebra carefully.]

(7 pts.) (ii) Suppose a vertical wall is to be built on top of the path C whose height is given by

$$f(x, y, z) = \frac{e^x y}{z}.$$

Find the surface area of this wall.

(7 pts) (iii) A force F is acting upon a particle traveling along the path C and $F(x, y, z) = (yz, e^x, z^2)$. What is the total work done $\int_C F \cdot d\mathbf{x}$ of F on the particle?