Math 205A Quiz 6, page 1 November 2, 2007 NAME *suggested solutions*.

1. Suppose **b** has components b_1, b_2, b_3, A is a 3x4 matrix and the augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$ is row equivalent to $\left[\begin{array}{cccc|c} 1 & 4 & 6 & 8 & b_1 - 2b_3 \\ 0 & 0 & 1 & 1 & -b_2 + 4b_3 \\ 0 & 0 & 0 & k & 3b_1 + 2b_2 + b_3 \end{array}\right]$.

1A. Suppose $k = 1$. What conditions (if any) must b_1, b_2 and b_3 satisfy in order for **b** to be in Col(A)? Explain! If $K=1$, the system is consistent for any b_1, b_2, b_3. So there are NO restrictions (No conditions) on b_1, b_2, b_3.

1B. So, if $k = 1$, is Col(A) all of $\mathbf{R}^3$? Explain!

yes, $Col(A) = \mathbb{R}^3$ since for any $\vec{b} \in \mathbb{R}^3$, $A\vec{x} = \vec{b}$ has a sol'n, i.e. $\vec{b}$ can be expressed as a LC of the columns of A.

1C. Suppose $k = 1$. Find vectors that span the nullspace of A. *Hint:* Think about the way we write the solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$ in "parametric form".

$$\begin{bmatrix} 1 & 4 & 6 & 8 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow A\vec{x} = \vec{0} \text{ has solutions } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

where x_2 is free. $\therefore$ all vectors in null(A) are multiples of the (single) vector $\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}$; this vector spans the nullspace.

1D. Suppose $k = 0$. What conditions (if any) must b_1, b_2 and b_3 satisfy in order for **b** to be in Col(A)? Explain! now $A\vec{x} = \vec{b}$ has a solution $\Leftrightarrow$ $\underbrace{3b_1 + 2b_2 + b_3 = 0}$ (otherwise the system is inconsistent)

1E. So, if $k = 0$, is Col(A) all of $\mathbf{R}^3$? Explain!

No. only vectors $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ satisfying THIS are in the column space now. $\left\{ \text{in particular, } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{R}^3 \text{ but } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \notin Col(A) \right.$

2. Let **F** be the vector space of all continuous functions $f : \mathbf{R} \to \mathbf{R}$, as discussed in class.

2A. Is the set $H = \{f \in \mathbf{F} \mid \text{the graph of } f \text{ passes through the point } (0, 3)\}$ closed under vector addition? Prove it or give a counterexample.

It's NOT closed. For (counter)example, let $f(x) = x+3$ and $g(x) = x^2+3$.
Since $f(0) = 3$, $f \in H$, and since $g(0) = 3$, $g \in H$. But $(f+g)(0) = f(0) + g(0) = 6$,
i.e. the graph of $f+g$ passes through $(0,6)$ instead of $(0,3)$, so $(f+g) \notin H$.
We've found two specific members of H whose sum is NOT in H

2B. Is the set $G = \{f \in \mathbf{F} \mid \text{the graph of } f \text{ passes through the point } (3, 0)\}$ closed under vector addition? Prove it or give a counterexample.

It IS CLOSED. Let f and g be any arbitrary members of G; we need to show $f+g \in G$.
So $f(3) = 0$ and $g(3) = 0$. Now, $(f+g)(3) = f(3) + g(3) = 0 + 0 = 0$,
So $f+g$ also passes through $(3,0)$ and thus $f+g \in G$.

2C. Which (if either) of H or G is a subspace of **F**? (H is NOT since it's not closed under addition, and doesn't even contain the 0 vector!!!)

only G (note: $\vec{0} \in G$ since the constant fn 0 passes through $(3,0)$; G is closed under addition AND it's easy to show it's closed under s.mult. too)