Math 205A Quiz 6, page 1 November 2, 2007 NAME______________

1. Suppose $\mathbf{b}$ has components b_1, b_2, b_3, A is a 3x4 matrix and the augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$ is row equivalent to $\left[\begin{array}{cccc|c} 1 & 4 & 6 & 8 & b_1 - 2b_3 \\ 0 & 0 & 1 & 1 & -b_2 + 4b_3 \\ 0 & 0 & 0 & k & 3b_1 + 2b_2 + b_3 \end{array}\right]$.

1A. Suppose $k = 1$. What conditions (if any) must b_1, b_2 and b_3 satisfy in order for $\mathbf{b}$ to be in $\mathrm{Col}(A)$? Explain!

1B. So, if $k = 1$, is $\mathrm{Col}(A)$ all of $\mathbf{R}^3$? Explain!

1C. Suppose $k = 1$. Find vectors that span the nullspace of A. *Hint:* Think about the way we write the solutions of the homogeneous equation $A\mathbf{x} = \mathbf{0}$ in "parametric form".

1D. Suppose $k = 0$. What conditions (if any) must b_1, b_2 and b_3 satisfy in order for $\mathbf{b}$ to be in $\mathrm{Col}(A)$? Explain!

1E. So, if $k = 0$, is $\mathrm{Col}(A)$ all of $\mathbf{R}^3$? Explain!

2. Let $\mathbf{F}$ be the vector space of all continuous functions $f : \mathbf{R} \to \mathbf{R}$, as discussed in class.

2A. Is the set $H = \{f \in \mathbf{F} \mid \text{the graph of } f \text{ passes through the point } (0,3)\}$ closed under vector addition? Prove it or give a counterexample.

2B. Is the set $G = \{f \in \mathbf{F} \mid \text{the graph of } f \text{ passes through the point } (3,0)\}$ closed under vector addition? Prove it or give a counterexample.

2C. Which (if either) of H or G is a subspace of $\mathbf{F}$?