

Math 106 Fall 2007

Test 2 (60 points)

Name: _____

Show all your work to receive full credit for a problem.

Do not use the calculator integral function.

When you use a formula from the table of integrals, mention the formula number and the value(s) of any constant(s) that you may need.

Round off your answers to four decimal places.

Include units in your answers wherever possible.

There are eight questions. Questions are printed on both sides of a page.

You may use any of the following facts:

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1}$$

$$\int u dv = uv - \int v du \qquad f(x) = \frac{1}{\sqrt{2\pi} s} \exp\left(\frac{-(x - m)^2}{2s^2}\right)$$

$$\int_1^\infty \frac{1}{x^p} dx \text{ converges for } p > 1 \text{ and diverges for } p \leq 1.$$

1. (6 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int (11z - 4) \sin(3z) dz$$

2. (6 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int \tan^2 x \sec^4 x \, dx$$

3. (7 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int \frac{x^2 - 5x + 13}{(x^2 + 3)(x - 2)} dx$$

4. (7 points) Evaluate the following definite integral exactly. In case of an improper integral, determine the convergence of the integral. Show clearly any limit computation you do. If the integral converges, find its value. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int_0^3 x \sqrt{25 - x^2} dx$$

5. (7 points) Evaluate the following definite integral exactly. In case of an improper integral, determine the convergence of the integral. Show clearly any limit computation you do. If the integral converges, find its value. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int_1^2 \frac{1}{x(\ln x)^2} dx$$

6. (7 points) Use comparisons to determine the convergence of the following integral.

$$\int_6^{\infty} \frac{3x^2 - x}{\sqrt{x^7 + 8x^5 + 10}} dx .$$

7. (6 points) The probability density function of the lifespan (in months) of certain batteries is given by $f(x) = .01e^{-.01x}$. What percentage of batteries are still functioning after 12 months? Note that the domain of f is $x \geq 0$ because we cannot have a negative lifespan.

8. (7 points) Let $f(x) = \sqrt[3]{x}$ and let $P_2(x)$ be the second-order Taylor polynomial for f based at $x_0 = 1$. What does Taylor's theorem imply about the maximum approximation error committed by P_2 over the interval $[1, 4]$? (Find the best possible value for K_{n+1} .)

9. (7 points) Suppose that the sixth-order Maclaurin polynomial for a function f is

$$M_6(x) = 2 + \frac{4x^2}{3} + \frac{x^3}{2} - \frac{7x^5}{24}.$$

Use this polynomial to answer the following questions.

- (a) Write the fifth-order Maclaurin polynomial for f .

- (b) Find $f^{(3)}(0)$.

- (c) Let $g(x) = xf(x)$. Write the third-order Maclaurin polynomial for g .