Math 205A Quiz 06 page 1 October 31, 2008 NAME suggested solution

1. Let $A = \begin{bmatrix} -5 & -6 & -6 \\ 5 & 6 & 6 \\ 1 & 0 & 2 \\ 1 & 3 & 0 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 7 \\ -7 \\ -7 \\ 7 \end{bmatrix}$.

1A. Find a set of vectors that spans Col(A).

$\left\{ \begin{bmatrix} -5 \\ 5 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 6 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 6 \\ 2 \\ 0 \end{bmatrix} \right\}$

[NB! it is a SET!] (it's written with { }'s)

1B. Is $\mathbf{b}$ in Col(A)? Explain. Does $A\vec{x} = \vec{b}$ have a soln? Finding RREF($[A|\vec{b}]$) gives

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -7 \\ 0 & 1 & -2/3 & 14/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ which says the system is consistent; YES $\vec{b} \in Col(A)$.

1C. Does Col(A)= $\mathbf{R}^4$? Explain.

No. The RREF above shows that for other $\vec{b}$'s, there will be inconsistencies, so not every $\vec{b}$ is in Col(A).

1D. Find a set of vectors that spans Nul(A).

The RREF above shows the soln's $\vec{v}_h$ of $A\vec{x} = \vec{0}$ are $\begin{cases} x_1 = -2x_3 \\ x_2 = 2/3\, x_3 \\ x_3 \text{ is free} \end{cases}$

so $\vec{v}_h$ = any/all L.C.'s of $\begin{bmatrix} -2 \\ 2/3 \\ 1 \end{bmatrix}$, or equivalently, of $\begin{bmatrix} -6 \\ 2 \\ 3 \end{bmatrix}$ $\therefore$ span$\left\{ \begin{bmatrix} -6 \\ 2 \\ 3 \end{bmatrix} \right\}$ = Nul(A)

1E. Suppose $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a linear transformation and A is its matrix. Is T one-to-one? Explain.

Since Ker(T) = Nul(A), and Nul(A) $\neq$ "just the zero vector", NO, T is NOT 1-1.

2. Let B be the 3 by 4 matrix all of whose entries are zeros.

2A. Col(B) is a subspace of $\mathbf{R}^p$ for what value of p? Explicitly, what is Col(B) in this example? Explain!

So $B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ L.C.'s of its columns are members of $\mathbb{R}^3$. However, any such L.C. is always $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\therefore$ Col(B) = $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = \{\vec{0}_3\}$

2B. Nul(B) is a subspace of $\mathbf{R}^d$ for what value of d? Explicitly, what is Nul(B) in this example? Explain!

Again, since any L.C. $a\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, this tells us any vector $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ is in Nul(B), that is, Nul(B) = $\mathbb{R}^4$ (and $d = 4$, by the way)