

NAME _____

I _____ II _____ III _____ IV _____ V _____ VI _____ VII _____ VIII _____ IX _____ TOTAL _____
(10) (10) (10) (10) (10) (10) (15) (10) (15) (100)

October 31
2008

Mathematics 106a
Calculus II
Examination #2

Mr. Haines

(10) I. Do ONE of these integrals, but NOT ALL OF THEM! You may choose the one you think is easiest:

$$\int x\sqrt{1-x^2} dx, \quad \int x^2\sqrt{1-x^2} dx, \quad \int x^3\sqrt{1-x^2} dx.$$

(10) II. Calculate the integral:

$$\int x^2 e^x dx.$$

(10) III. Does $\int_{\frac{\pi}{2}}^{\infty} \frac{1}{x - \sin x} dx$ converge? Justify your answer.

(10) IV. Evaluate $\int_0^{\infty} e^{-2x} dx$.

(10) V. Evaluate the integral $\int_1^{\infty} \frac{1}{x^{10}} dx$. If the value is a number, give that number. If the integral diverges, explain why.

10) VI. Evaluate the integral $\int_0^1 \frac{1}{x^{10}} dx$. If the value is a number, give that number. If the integral diverges, explain why.

(15) VII. Find the integral

$$\int \frac{3x^2 - 2x}{x^2(x-1)} dx$$

10) VIII. Give the third order Maclaurin polynomial ($x_0 = 0$) for

$$f(x) = \frac{1}{1+x} = (1+x)^{-1}.$$

(15) IX. The second order Maclaurin polynomial ($x_0 = 0$) for $f(x) = \ln(1+x)$ is

$$P_2(x) = x - \frac{x^2}{2}.$$

A. Calculate $|f^{(3)}(x)|$.

B. If $I = [-0.5, 0.5]$, find an upper bound K_3 for $|f^{(3)}(x)|$ on I .

C. Taylor's Theorem says that $|f(x) - P_2(x)| \leq \frac{K_3}{3!} |x - x_0|^3$ for all values of x in an interval I containing x_0 . Calculate the maximum approximate error for values of x in I , i.e. the maximum value of $\frac{K_3}{3!} |x - x_0|^3$ on I .