Here are some facts:

Let $A = \begin{bmatrix} -4 & -8 & -16 & 8 & -10 & -14 \\ 6 & 19 & 31 & 16 & 26 & 20 \\ 5 & 16 & 26 & 14 & 20 & 7 \\ 3 & 10 & 16 & 10 & 17 & 26 \end{bmatrix}$, and let

$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 30 \\ 37 \\ 0 \\ 79 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ -7 \\ 3 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -13 \\ 24 \\ 4 \\ 3 \\ -25 \\ 5 \end{bmatrix}$

Let the columns of A be $\mathbf{c}_1 \ldots \mathbf{c}_6$, and let the columns of rref(A) be $\mathbf{k}_1 \ldots \mathbf{k}_6$. Let $R =$rref(A).

Fact 1. The rref of $[A \mid I_4]$ is $\left[\begin{array}{cccccc|cccc} 1 & 0 & 2 & -10 & 0 & 7 & 0 & 8 & -7 & -4 \\ 0 & 1 & 1 & 4 & 0 & -8 & 0 & -25/9 & 8/3 & 10/9 \\ 0 & 0 & 0 & 0 & 1 & 5 & 0 & 2/9 & -1/3 & 1/9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 12 & -10 & -6 \end{array}\right]$

Fact 2. The matrix product $\begin{bmatrix} -4 & -8 & -16 & 8 & -10 & -14 \\ 6 & 19 & 31 & 16 & 26 & 20 \\ 5 & 16 & 26 & 14 & 20 & 7 \\ 3 & 10 & 16 & 10 & 17 & 26 \end{bmatrix} \begin{bmatrix} -13 & 3 \\ 24 & 4 \\ 4 & -7 \\ 3 & 3 \\ -25 & 2 \\ 5 & 3 \end{bmatrix}$ is $\begin{bmatrix} 0 & 30 \\ 0 & 37 \\ 0 & 0 \\ 0 & 79 \end{bmatrix}$

Fact 3. Finally, the rref of

$$\left[\begin{array}{cccccc|c} -4 & -8 & -16 & 8 & -10 & -14 & 30 \\ 6 & 19 & 31 & 16 & 26 & 20 & 37 \\ 5 & 16 & 26 & 14 & 20 & 7 & 0 \\ 3 & 10 & 16 & 10 & 17 & 26 & 79 \end{array}\right] \text{ is } \left[\begin{array}{cccccc|c} 1 & 0 & 2 & -10 & 0 & 7 & -20 \\ 0 & 1 & 1 & 4 & 0 & -8 & -15 \\ 0 & 0 & 0 & 0 & 1 & 5 & 17 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The first question is *here:*

1. What does it mean (ie, what's the definition) for a set $\mathcal{B}$ of vectors $\{\mathbf{v}_1 \ldots \mathbf{v}_p\}$ to be a *basis* of a subspace H of a vector space.

2. Let A, $\mathbf{b}$, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be as on page one of this Quiz.

(2A) What conditions, if any, do the entries $b_1, \ldots b_4$ of $\mathbf{b}$ have to satisfy in order for $\mathbf{b}$ to be in Col(A)?

(2B) Verify that $\mathbf{u}$ is in Col(A) according to the conditions in (2A).

(2C) Is $\mathbf{k}_1$ in Col(A)? Explain.

(2D) Are any column vectors of A in the column space of R? Explain.

(2E) Which column vectors of A form a basis of Col(A)? (write your answer in terms of the $\mathbf{c}_i$'s)

(2F) Express $\mathbf{u}$ as a linear combination (LC) of the basis vectors in (2E).
(write your answer in terms of the $\mathbf{c}_i$'s, eg "$3\mathbf{c}_2 + 4\mathbf{c}_5$")

(2G) What LC is $3\mathbf{c}_1 + 4\mathbf{c}_2 - 7\mathbf{c}_3 + 3\mathbf{c}_4 + 2\mathbf{c}_5 + 3\mathbf{c}_6$? and how did you find it? (eg, "I used fact 7" or "I used my calculator to compute..." or "I computed [give the expression] by hand", etc)

(2H) What LC is $-13\mathbf{c}_1 + 24\mathbf{c}_2 + 4\mathbf{c}_3 + 3\mathbf{c}_4 + -25\mathbf{c}_5 + 5\mathbf{c}_6$? and how did you find it?

(2I) Does the result of (2H) say that $\mathbf{w}$ is or is not in Nul(A)?

(2J) Find a basis for Nul(A).

(2K) Express $\mathbf{w}$ as a LC of the basis vectors from (2J).