MATH 106A - CALCULUS II  
FALL 2005

QUIZ 6

NAME:

Show ALL your work CAREFULLY.

(a) Find the second-order Maclaurin’s polynomial $M_2(x)$ for the function $f(x) = \sin(x^2)$.

Maclaurin’s polynomials are simply Taylor’s polynomials with $x_0 = 0$. To find $M_2(x)$, we first evaluate the derivatives of $f(x) = \sin(x^2)$. By the chain rule, $f'(x) = \cos(x^2) \cdot 2x$. Together with the product rule,

$$f''(x) = 2\cos(x^2) - 4x^2 \sin(x^2).$$

It follows that $f(0) = 0$, $f'(0) = 0$, and $f''(0) = 2\cos(0) - 0 = 2$. Thus, $M_2(x) = 0 + 0x + \frac{2}{2!}x^2 = x^2$.

(b) Use the Taylor’s theorem to give an upper bound for the error committed by using $M_2(x)$ to estimate $f(x)$ for $-1 \leq x \leq 1$. [Do your differentiation carefully.]

Taylor’s theorem asserts that $|f(x) - M_2(x)| \leq \frac{K_3}{3!}|x|^3$ where $K_3$ is a constant such that $|f'''(x)| \leq K_3$. By differentiating $f''(x)$ from (1), we get

$$f'''(x) = -2\sin(x^2) \cdot 2x - 4[2x \cdot \sin(x^2) + x^2 \cdot \cos(x^2) \cdot 2x]$$

$$= -[12x \sin(x^2) + 8x^3 \cos(x^2)].$$

Since $|\sin(x^2)| \leq 1$, $|\cos(x^2)| \leq 1$, and $|x| \leq 1$ for $-1 \leq x \leq 1$, it follows that $|f'''(x)| \leq 12 + 8 = 20$ so that we let $K_3 = 20$. Hence, we conclude that $|f(x) - M_2(x)| \leq \frac{20}{3!} = \frac{10}{3}$.

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