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I___II___III___IV___V___VI___VII___VIII___ IX___X___XI___TOTAL ___________

October 27 2004 | Mathematics 205 Linear Algebra Examination #2 | Mr. Haines

(10) I. Suppose $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 0 & -5 \\ 3 & -7 & 8 & -5 & 3 & 9 \\ 3 & -9 & 12 & -9 & 3 & 15 \end{bmatrix}$

A. If col A is a subspace of $\Re^m$, what is the value of m ?

B. If nul A is a subspace of $\Re^m$, what is the value of m ?

(5) II. Give an example of a two-dimensional subspace of $\Re^4$. Use correct mathematical notation to describe it.

(20) III. If $A = \begin{bmatrix} 1 & -1 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

A. Find a basis for Col A.

B. Find a basis for Nul A.

C. What is the dimension of Col A?

D. What is the dimension of Nul A?

E. What is the rank of A?

(10) IV. If $T : \Re^2 \rightarrow \Re^2$ reflects points through the line $x_1 = x_2$,

A. What is the determinant of the standard matrix of the linear transformation T?

B. How many pivot positions does this matrix have?

(5) V. Because $A A^{-1} = I$, it follows that (det A)(det A^{-1}) = det I . If $A = \begin{bmatrix} 7 & 1 & 1 \\ 0 & 2 & 5 \\ -7 & -1 & 1 \end{bmatrix}$,

calculate det A^{-1}.

(5) VI. Suppose

that B is obtained from A by interchanging the first two rows of A,

and that det (A) = det (B).

What is the value of det (A) ?

(5) VII. Give an example of a matrix A whose null space, Nul A, is a straight line in $\Re^3$.

(10) VIII. Compute the area of the parallelogram whose vertices are the points
(4, 5), (1, 1), (2, 4), and (3, 2).

(10) IX. Suppose $AB = \begin{bmatrix} 14 & -2 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix}$. Find A.

(15) X. Consider the production model $\mathbf{x} = C\mathbf{x} + \mathbf{d}$ for an economy with two sectors, where

$$C = \begin{bmatrix} 0.0 & 0.5 \\ 0.6 & 0.3 \end{bmatrix}.$$

A. Compute the matrix $I - C$.

B. Compute the inverse of the matrix $I - C$.

C. Use this inverse to determine the production level necessary to satisfy the final demand $\mathbf{d} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

(5) XI. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

________ 1. If A is a 2×2 matrix with a zero determinant, then one column of A is a multiple of the other.

________ 2. It is possible to have two matrices A and B that are invertible, but their product is not invertible.

________ 3. If $AB = AC$, then $B = C$ for all matrices A, B, and C.

________ 4. If H is a subspace of $\Re^m$ and $\dim(H) = 4$, then m must be greater than or equal to 4.

_______ 5. If H is a subspace of $\Re^m$ and $\dim H = 4$, then H could have a basis with 2 elements.