Math 205A Quiz 5, page 1 October 26, 2007 NAME Suggested solutions

1. Let $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $A = \begin{bmatrix} 2 & 3 & 0 & 4 \\ 4 & 9 & 1 & 8 \\ -10 & t & 3 & -19 \end{bmatrix}$. Suppose $A = LU$.

1A. What is the value of t? (What expression did you evaluate to find it?)

t is "(row 3 of L) $*$ (col 2 of U)" $= -5\cdot 3 + 3\cdot 3 + 1\cdot 0 = -15+9 = \boxed{-6}$

1B. Use the method of LU decomposition as discussed in class to solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ -37 \end{bmatrix}$.

Show all your steps. Write the solution as $\mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}_h$ gives the solution(s) of the corresponding homogeneous equation.

$A\vec{x} = \vec{b} \Rightarrow$

$LU\vec{x} = \vec{b} \Rightarrow$

$L\vec{y} = \vec{b} \Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -37 \end{bmatrix}$$

$y_1 = 4$

$2y_1 + y_2 = 2 \Rightarrow 8 + y_2 = 2 \Rightarrow y_2 = -6$

$-5y_1 + 3y_2 + y_3 = -37 \Rightarrow -20 - 18 + y_3 = -37 \Rightarrow y_3 = -37 + 38 = 1$

so $\vec{y} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$ and $\vec{y} = U\vec{x}$; solve $\begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix}$

now, $x_4 = 1$

$x_3 =$ free

$3x_2 + x_3 = -6 \Rightarrow$

$x_2 = \frac{-6 - x_3}{3} = -2 - \frac{1}{3}x_3$

$2x_1 + 3x_2 + 4x_4 = 4 \Rightarrow$

$x_1 = \frac{1}{2}(4 - 3x_2 - 4x_4)$

$= \frac{1}{2}(4 - 3(-2 - \frac{1}{3}x_3) - 4)$

$= \frac{1}{2}(6 + x_3) = 3 + x_3/2$

so $\vec{x} = \begin{bmatrix} 3 + x_3/2 \\ -2 - x_3/3 \\ x_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1/2 \\ -1/3 \\ 1 \\ 0 \end{bmatrix}$

1C. The augmented matrix $[A|\mathbf{b}]$ is row equivalent to $\left[\begin{array}{cccc|c} 1 & 0 & -1/2 & 0 & 3 \\ 0 & 1 & 1/3 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$. Use this to again write the solution of $A\mathbf{x} = \mathbf{b}$ as $\mathbf{p} + \mathbf{v}_h$. How does this solution compare with the one from 1B (is the particular solution the same?)

here $\vec{x} = \begin{bmatrix} 3 + \frac{1}{2}x_3 \\ -2 - \frac{1}{3}x_3 \\ 0 + 1\,x_3 \\ 1 + 0\,x_3 \end{bmatrix}$ where x_3 is free, that is, $\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1/2 \\ -1/3 \\ 1 \\ 0 \end{bmatrix}$; same as 1b's particular soln.

1D. What one elementary row operation changes A into $\begin{bmatrix} 2 & 3 & 0 & 4 \\ 4 & 9 & 1 & 8 \\ 0 & * & 3 & 1 \end{bmatrix}$? (Describe it: "Row 3 is replaced by ...")

R_3 is replaced by $R_3 + 5R_1$

1E. What is the value of $*$ after this operation?

($t + 5\cdot$ row 1 element)

in A, t is -6 and so $* = -6 + 5\cdot 3$
$= -6 + 15 = 9$

1F. What elementary matrix E corresponds to, or, carries out this operation, if you compute EA?

Do to I_3 what you want to do to A, that is, replace its third row by $R_3 + 5R_1$. Since $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

this yields $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$.

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2. Let $B = \begin{bmatrix} 9 & 7 & 4 & -9 \\ 0 & 0 & 5 & 8 \\ 0 & 0 & 6 & 10 \\ 11 & 0 & 21 & 90 \end{bmatrix}$.

2a. Find the determinant of B. Expand using any row or column you like, but look at all those 0's in the second column!

$$\text{So } \det(B) = -7 \begin{vmatrix} 0 & 5 & 8 \\ 0 & 6 & 10 \\ 11 & 21 & 90 \end{vmatrix} + 0\,|?| + 0\,|?| + 0\,|?|$$

$$= -7 \cdot 11 \begin{vmatrix} 5 & 8 \\ 6 & 10 \end{vmatrix} = -7 \cdot 11 \cdot (50 - 48) = -7 \cdot 11 \cdot 2 = -154$$

2b. Find $\det(B^T)$.

$$\det(B^T) = \det(B) = -154$$

2c. Find $\det(B^{-1})$.

$$\det(B^{-1}) = \frac{1}{\det(B)} = \frac{-1}{154}$$

(provided $\det(B) \neq 0$)