1. Let $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $A = \begin{bmatrix} 2 & 3 & 0 & 4 \\ 4 & 9 & 1 & 8 \\ -10 & t & 3 & -19 \end{bmatrix}$. Suppose $A = LU$.

1A. What is the value of t? (What expression did you evaluate to find it?)

1B. Use the method of LU decomposition as discussed in class to solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ -37 \end{bmatrix}$. Show all your steps. Write the solution as $\mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}_h$ gives the solution(s) of the corresponding homogeneous equation.

1C. The augmented matrix $[A|\mathbf{b}]$ is row equivalent to $\left[\begin{array}{cccc|c} 1 & 0 & -1/2 & 0 & 3 \\ 0 & 1 & 1/3 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$. Use this to again write the solution of $A\mathbf{x} = \mathbf{b}$ as $\mathbf{p} + \mathbf{v}_h$. How does this solution compare with the one from 1B (is the particular solution the same?)

1D. What one elementary row operation changes A into $\begin{bmatrix} 2 & 3 & 0 & 4 \\ 4 & 9 & 1 & 8 \\ 0 & * & 3 & 1 \end{bmatrix}$? (Describe it: "Row 3 is replaced by ...")

1E. What is the value of $*$ after this operation?

1F. What elementary matrix E corresponds to, or, carries out this operation, if you compute EA?

2. Let $B = \begin{bmatrix} 9 & 7 & 4 & -9 \\ 0 & 0 & 5 & 8 \\ 0 & 0 & 6 & 10 \\ 11 & 0 & 21 & 90 \end{bmatrix}$.

2a. Find the determinant of B. Expand using any row or column you like, but look at all those 0's in the second column!

2b. Find $\det(B^T)$.

2c. Find $\det(B^{-1})$.