 NAME Suggested Soln's

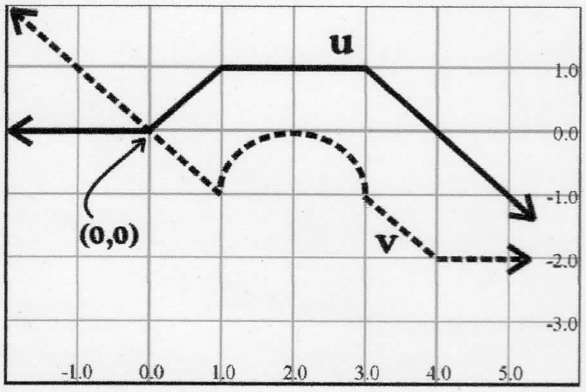
1. Suppose V is a vector space and H is a subset of V. What are the three properties that H must satisfy in order to be a sub*space* of V? (List in the same order as we have in class)

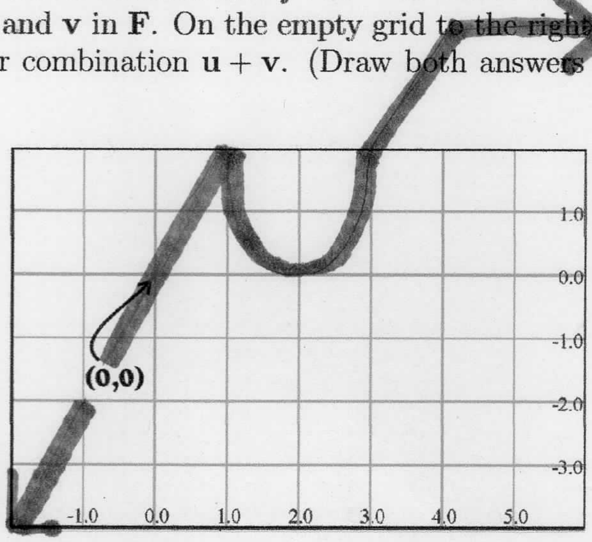
(1) $\vec{0} \in H$.

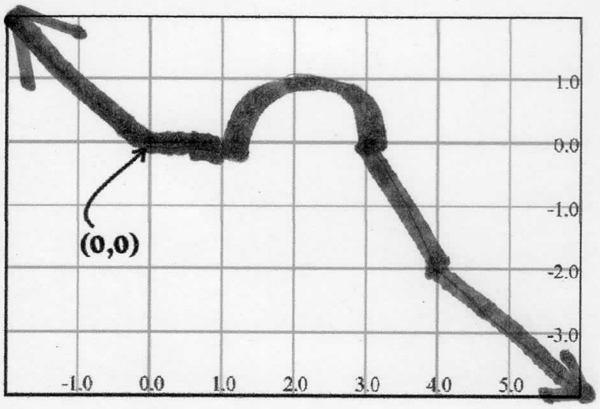
(2) If $\vec{u}$ and $\vec{v}$ belong to H, so does $\vec{u}+\vec{v}$.

(3) If $\vec{u} \in H$ and $\alpha \in \mathbb{R}$, then $\alpha\vec{u} \in H$.

2. As in class, let **F** be the vector space whose members are functions $f : \mathbf{R} \to \mathbf{R}$ which are continuous. The figure below shows the graphs of two vectors **u** and **v** in **F**. On the empty grid to the right, show the graph of $-2\mathbf{v}$. On the bottom grid, show the linear combination $\mathbf{u} + \mathbf{v}$. (Draw both answers with solid lines)





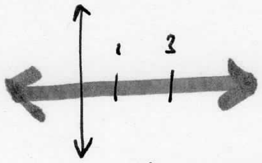


3. Let H be the subset of all vectors in **F** whose graph is horizontal from $x = 1$ to $x = 3$.

3a. Is either of **u** or **v** from problem (**2**) in H? Explain both decisions.

The graph of $\vec{u}$ IS horizontal from 1 to 3, so $\vec{u} \in H$.
But the graph of $\vec{v}$ is curved ('it's the top half of a circle of radius 1)
from 1 to 3, so $\vec{v} \notin H$.

3b. For each of the 3 properties in problem (**1**), decide if the property holds for H or does not. If it holds, explain why; if it does not, then give a counterexample (in pictures if you wish).

(1) The $\vec{0}$ vector (in $\mathbb{F}$) is  , which is horizontal EVERYWHERE, and in particular from 1 to 3. $\therefore$ $\vec{0}$ is in H.

(2) If two functions $\vec{u}$ and $\vec{v}$ are constant from 1 to 3, then so is their sum, so $\vec{u}+\vec{v} \in H$ when $\vec{u}$ & $\vec{v}$ are.

(3) If the graph of $\vec{u}$ is horizontal from 1 to 3, having a value of, say, c, then the graph of $\alpha\vec{u}$ has a value of αc from 1 to 3, that is, the graph of $\alpha\vec{u}$ is also horizontal.

4. Let $\mathcal{S}$ be the "script-S" vector space of all infinite sequences of real numbers we've discussed in class. Let H be the subset of $\mathcal{S}$ consisting of all sequences which change sign from term-to-term, that is, if **s** is a sequence in H, and some term of **s** is positive, the one after it is negative, and vice-versa. *By default*, we will also automatically add the sequence $\mathbf{0} = (0, 0, 0, \ldots)$ to H.

4a. Is $\mathbf{s}_1 = (1, -2, 3, -4, 5, -6, \ldots)$ in H? Yes, the terms change sign from + to − to + to − "forever"

4b. Is $\mathbf{s}_2 = (3, 0, -3.1, 0, 3.14, 0, -3.141, \ldots)$ in H? No. Since the 1st term is positive, the second should be negative if $\vec{s}_2 \in H$. But 0 is NOT negative, so $\vec{s}_2 \notin H$.

For each of the 3 properties in problem (**1**), decide if the property holds for H or does not. If it holds, explain why; if it does not, then give a counterexample using specific sequences.

(1) $\vec{0}$ is in H "by default" (it's part of the definition of H)

(2) no. For example, $\vec{u} = (-1, 2, -1, 2, \ldots)$ and $\vec{v} = (2, -1, 2, -1, \ldots)$ both are in H, but their sum, $\vec{u}+\vec{v}$ is $(1, 1, 1, 1, \ldots)$ which is NOT in H.

(3) yes. Let $\vec{u}$ be in H. If $\alpha > 0$, then $\alpha\vec{u}$ is a sequence whose terms have the same signs as those in H; if $\alpha < 0$ then $\alpha\vec{u}$ is a sequence whose terms are opposite in sign; in either case $\alpha\vec{u} \in H$. If $\alpha = 0$ =OR= $\vec{u} = \vec{0}$, then $\alpha\vec{u} = \vec{0} \in H$ by default.