Math 105 - Quiz 8 - October 22, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (10 pts.) Let \( f(x) = 3x + \sin x \). On what subinterval(s) in \([-5, 5]\) is \( f(x) \) concave up? Find all inflection points of \( f(x) \) in the interval \([-5, 5]\).

\[
\frac{d}{dx}(3x + \sin x) = 3 + \cos x.
\]

\[
\frac{d^2}{dx^2}(3x + \sin x) = -\sin x.
\]

In \([-5, 5]\), \(-\sin x = 0\) at \( x = -\pi, 0, \pi \).

\[
\frac{d^3}{dx^3}(3x + \sin x) = -\cos x.
\]

\[
\begin{array}{c|c}
  x & f''(x) \\
  \hline
  -\pi & -1 \\
  0 & 0 \\
  \pi & -1 \\
\end{array}
\]

\( f''(x) > 0 \) on \((-\pi, 0)\) and \((\pi, 5]\).

So \( f(x) \) is concave up on \((-\pi, 0)\) and \((\pi, 5]\).

\( f(x) \) is concave up on the remaining intervals, meaning \( f(x) \) switches concavity at \( x = -\pi, x = 0, \) and \( x = \pi \), which are the inflection points.

2. (10 pts.) Let \( g(x) = \frac{\ln x}{2x^3} + e^x \cos x \). Find \( g'(x) \).

Quotient rule:

\[
\frac{d}{dx}\left(\frac{\ln x}{2x^3}\right) = \frac{2x^3 \cdot \frac{d}{dx}(\ln x) - (\ln x) \cdot \frac{d}{dx}(2x^3)}{(2x^3)^2} = \frac{2x^3 \cdot \frac{1}{x} - (\ln x) \cdot 6x^2}{4x^6}
\]

\[
= \frac{2x - 6x \ln x}{4x^6}
\]

Product rule:

\[
\frac{d}{dx}(e^x \cos x) = \frac{d}{dx}(e^x) \cdot \cos x + e^x \frac{d}{dx}(\cos x) = e^x \cos x + e^x (-\sin x) = e^x \cos x - e^x \sin x.
\]

So, \( g'(x) = \frac{1 - 3 \ln x}{2x^4} + e^x \cos x - e^x \sin x \).