 Name Suggested solutions

1. Let $A = \begin{bmatrix} 1 & a & 2 \\ b & 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 \\ 4 & 2 \\ 8 & 1 \end{bmatrix}$.

(1A) Find the matrix product AB.

$$= \begin{bmatrix} 1\cdot c + a\cdot 4 + 2\cdot 8 & 1\cdot 3 + a\cdot 2 + 2\cdot 1 \\ b\cdot c + 3\cdot 4 + 5\cdot 8 & b\cdot 3 + 3\cdot 2 + 5\cdot 1 \end{bmatrix} = \begin{bmatrix} c + 4a + 16 & 2a + 5 \\ bc + 52 & 3b + 11 \end{bmatrix}$$

(1B) Suppose the first two entries in the first row of this product are 0 and -1 respectively. Find c.

so $c + 4a + 16 = 0$

and $2a + 5 = -1 \Rightarrow 2a = -6 \Rightarrow a = -3 \Rightarrow c + 4(-3) + 16 = 0 \Rightarrow c - 12 + 16 = 0$

$\Rightarrow c + 4 = 0 \Rightarrow c = -4$

2. Suppose $A = \begin{bmatrix} m & n \\ s & t \end{bmatrix}$. What is A^{-1}? Under what conditions on m, n, s, t is A a singular matrix?

$A^{-1} = \frac{1}{mt - sn} \begin{bmatrix} t & -n \\ -s & m \end{bmatrix}$, provided $mt - sn \neq 0$.

A is singular means in fact, that $mt - sn$ DOES equal 0, and hence, A^{-1} does not exist.

3. Suppose A is a 2x2 matrix which row-reduces to I_2 after the following row operations are applied in this order to A:

(step 1) Row 1 of A is replaced with (row 1 + 4 row 2).

(step 2) The second row of the resulting matrix is divided by 2.

for ref: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(step 3) In the matrix resulting from step (2), the second row is replaced with (row 2 + 3 row 1).

(3A) What are the three elementary matrices E_1, E_2, E_3 corresponding to the three steps 1, 2 and 3, respectively?

$E_1 = \boxed{\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}}$ $E_2 = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}}$ $E_3 = \boxed{\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}}$

(3B) The product of E_1, E_2, E_3, in some order, gives A^{-1}. What is that order? (eg, "$E_2E_1E_3$"?)

$E_3E_2E_1A = I_2 \Rightarrow E_3E_2E_1$ is A^{-1}

(3C) From (3B) what is A^{-1}?

putting E_1 E_2 & E_3 into the calculator as matrices [A] [B] & [C] respectively, then asking for the matrix product [C][B][A] yields $A^{-1} = \begin{bmatrix} 1 & 4 \\ 3 & 12.5 \end{bmatrix}$

NB: [A] means $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$, not the "A" of this problem!

(3D) What is A? Explain how you found it (there is more than one way to find it).

1) one way: RREF$\left(\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 12.5 & 0 & 1 \end{array}\right]\right)$ yields $\left[\begin{array}{cc|cc} 1 & 0 & 25 & -8 \\ 0 & 1 & -6 & 2 \end{array}\right]$

(A is the A of this problem [A] is the 1st matrix variable in the calculator)

2) OR, use problem 2 above: $\frac{1}{1\cdot 12.5 - 3\cdot 4}\begin{bmatrix} 12.5 & -4 \\ -3 & 1 \end{bmatrix} = \frac{1}{12.5 - 12}\begin{bmatrix} * & * \\ * & * \end{bmatrix} = \frac{1}{1/2}\begin{bmatrix} * & * \\ * & * \end{bmatrix} = 2\begin{bmatrix} 12.5 & -4 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 25 & -8 \\ -6 & 2 \end{bmatrix}$

3) OR use the $^{-1}$ on your calculator, applied to the result of (3C).