

1. Let $A = \begin{bmatrix} 1 & a & 2 \\ b & 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} c & 3 \\ 4 & 2 \\ 8 & 1 \end{bmatrix}$.

(1A) Find the matrix product AB .

(1B) Suppose the first two entries in the first row of this product are 0 and -1 respectively. Find c .

2. Suppose $A = \begin{bmatrix} m & n \\ s & t \end{bmatrix}$. What is A^{-1} ? Under what conditions on m, n, s, t is A a singular matrix?

3. Suppose A is a 2×2 matrix which row-reduces to I_2 after the following row operations are applied in this order to A :

(step 1) Row 1 of A is replaced with (row 1 + 4 row 2).

(step 2) The second row of the resulting matrix is divided by 2.

(step 3) In the matrix resulting from step (2), the second row is replaced with (row 2 + 3 row 1).

(3A) What are the three elementary matrices E_1, E_2, E_3 corresponding to the three steps 1, 2 and 3, respectively?

$$E_1 = \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$E_2 = \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$E_3 = \begin{array}{|c|} \hline \\ \hline \end{array}$$

(3B) The product of E_1, E_2, E_3 , in some order, gives A^{-1} . What is that order? (eg, " $E_2E_1E_3$ "?)

(3C) From (3B), what is A^{-1} ?

(3D) What is A ? Explain how you found it (there is more than one way to find it).