

Name: _____

Mathematics 205
Exam I
October 15, 2010

Problem	Possible	Actual
1	15	
2	15	
3	6	
4	8	
5	12	
6	14	
7	15	
8	15	
Total	100	

You must show all work to receive credit.
No electronic devices other than calculators are permitted.
Give exact answers (such as $\ln 5$ or e^2) unless requested otherwise.

1. Suppose $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & 0 & k \\ 2 & 5 & 6 & 0 \\ 0 & 6 & 4 & 2 \end{bmatrix}$.

(a) If $k = 1$, what is $\det(B)$?

(b) What value of k makes B not invertible?

2. (a) What are the properties of a subspace?

(b) Let $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ where } x_1 + x_2 \geq x_3 \right\}$. Is S a subspace of \mathbb{R}^3 ?

3. What are the properties required for a mapping to be a linear transformation?

4. If a mapping is defined by $T(\vec{x}) = A\vec{x}$ where A is a matrix, explain why T is a linear transformation.

5. Consider $P = \{a_0 + a_1x + a_2x^2, a_i \in \mathbb{R}\}$, the set of all constant, linear and quadratic polynomials over \mathbb{R} . We may regard an element of P as a vector in \mathbb{R}^3 .

For example, $\pi + 3x + x^2$, $4 + 8x$, and $a_0 + a_1x + a_2x^2$ have the vectors $\begin{bmatrix} \pi \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$.

- (a) Show that the derivative, $\frac{d}{dx}$, is a linear transformation of elements in P .

- (b) What is the matrix of the transformation of the derivative. (Hint: The vector $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ corresponds to polynomial x . What is the derivative of x and what is the vector that corresponds to that answer?)

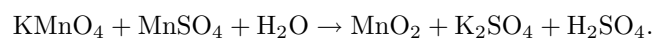
6. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) What does it mean for a mapping to be onto? Is $T(\vec{x}) = A\vec{x}$ onto?

(b) What does it mean for a mapping to be one-to-one? Is $T(\vec{x}) = A\vec{x}$ one-to-one?

7. A matrix A is invertible. Write five equivalent statements from the Invertible Matrix Theorem.

8. The following reaction between potassium permanganate (KMnO_4) and manganese sulfate in water produces manganese dioxide, potassium sulfate, and sulfuric acid:



Balance the chemical equation using techniques learned in this class.