1. Let $C = \begin{bmatrix} 3 & 7 & w & 2 \\ 9 & 5 & 1 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix}$ let $D = \begin{bmatrix} 8 & 3 \\ 1 & 5 \\ 2 & 0 \\ 0 & 11 \end{bmatrix}$. Suppose the 1-1 entry of the product CD is 51.

Find each of the following values. If the value doesn't exist write "DNE" in the box, and give the reason.

a. The number of entries in CD. $\boxed{6}$

(Size is "(3×4) by (4×2)" $\Rightarrow 3\times2 \therefore 6$)

b. The 2-1 entry of CD. $\boxed{79}$

$9\cdot8+5\cdot1+1\cdot2+3\cdot0 = 72+5+2+0$

c. The 4-3 entry of CD. $\boxed{\text{DNE}}$

DNE since CD is 3×2

d. The value of w. $\boxed{10}$

$3\cdot8+7\cdot1+w\cdot2+2\cdot0=51$

$2w=51-24-7=20$

e. The 3-2 entry of C^T. $\boxed{1}$

is the 2-3 entry of C

f. The 2-3 entry of $(DC)^T$ $\boxed{\text{DNE}}$

The product DC is undefined. $\therefore$ DNE

g. The 2-3 entry of $(CD)^T$ $\boxed{76}$

is the 3-2 entry of CD: $0\cdot3+2\cdot5+4\cdot0+6\cdot11=76$

h. The size of the matrix product CC^T. $\boxed{3\times3}$

(3×4) by (4×3)

i. The 2-1 entry of $D^T D$. $\boxed{29}$

$\begin{bmatrix} 8 & 1 & 2 & 0 \\ 3 & 5 & 0 & 11 \end{bmatrix} D \therefore\ 3\cdot8+5\cdot1+0\cdot2+11\cdot0=29$

j. The 1-2 entry of $D^T D$. $\boxed{29}$

note the same #'s are involved! ("different orders")

2. Let $A = \begin{bmatrix} 4 & 22 & 16 \\ 10 & 58 & 51 \\ -2 & -5 & 15 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 94 \\ 228 \\ -63 \end{bmatrix}$.

2a. Find A^{-1} on your calculator using any method you like. *Give your answer in fractions!*

A^{-1} is: $\begin{bmatrix} 375/4 & -205/6 & 97/6 \\ -21 & 23/3 & -11/3 \\ 11/2 & -2 & 1 \end{bmatrix}$ (done by setting TI's [A] to A, then finding $[A]^{-1}$)

2b. Solve $A\mathbf{x} = \mathbf{b}$. *Give your answer in fractions!*

$\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}$ (by setting [B] to $\begin{bmatrix} 94 \\ 228 \\ -63 \end{bmatrix}$ in T.I., then computing $[A]^{-1}[B]$)

3. Let $A \in M_{n\times n}$. Give four statements which are equivalent to the statement "A is invertible". (From the invertible matrix theorem). Note that "A^{-1} exists" is *not* one of them.

Hints: span, linearly independent, RREF, solutions of $A\mathbf{x} = \mathbf{0}$, pivot, solutions of $A\mathbf{x} = \mathbf{b}$, associated linear transformation, ... The hints reminded us of:

3a. The columns of A span $\mathbb{R}^n$.

3b. The columns of A form a linearly independent set.

3c. $\text{RREF}(A) = I_n$.

3d. The only solution of $A\vec{x}=\vec{0}$ is the trivial solution.

3e. A has n pivot positions.

3f. $A\vec{x}=\vec{b}$ has at least one solution for each $\vec{b}\in\mathbb{R}^n$.