

1. Let  $C = \begin{bmatrix} 3 & 7 & w & 2 \\ 9 & 5 & 1 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix}$  let  $D = \begin{bmatrix} 8 & 3 \\ 1 & 5 \\ 2 & 0 \\ 0 & 11 \end{bmatrix}$ . Suppose the 1-1 entry of the product  $CD$  is 51.

Find each of the following values. If the value doesn't exist write "DNE" in the box, and give the reason.

a. The number of entries in  $CD$ .

b. The 2-1 entry of  $CD$ .

c. The 4-3 entry of  $CD$ .

d. The value of  $w$ .

e. The 3-2 entry of  $C^T$ .

f. The 2-3 entry of  $(DC)^T$

g. The 2-3 entry of  $(CD)^T$

h. The size of the matrix product  $CC^T$ .

i. The 2-1 entry of  $D^T D$ .

j. The 1-2 entry of  $D^T D$ .

2. Let  $A = \begin{bmatrix} 4 & 22 & 16 \\ 10 & 58 & 51 \\ -2 & -5 & 15 \end{bmatrix}$  and let  $\mathbf{b} = \begin{bmatrix} 94 \\ 228 \\ -63 \end{bmatrix}$ .

2a. Find  $A^{-1}$  on your calculator using any method you like. *Give your answer in fractions!*

2b. Solve  $A\mathbf{x} = \mathbf{b}$ . *Give your answer in fractions!*

3. Let  $A \in M_{n \times n}$ . Give four statements which are equivalent to the statement " $A$  is invertible". (From the invertible matrix theorem). Note that " $A^{-1}$  exists" is *not* one of them.

*Hints:* span, linearly independent, RREF, solutions of  $A\mathbf{x} = \mathbf{0}$ , pivot, solutions of  $A\mathbf{x} = \mathbf{b}$ , associated linear transformation, ....

3a.

3b.

3c.

3d.