Math 205A Quiz 4, page 1 October 12, 2007 NAME SUGGESTED SOLUTIONS

1. Let $A = \begin{bmatrix} 5 & -6 & 1 \\ 2 & -4 & 7 \\ 1 & 0 & 2 \\ -2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -20 & 30 & -10 \\ 5 & 10 & 20 & 30 \\ 15 & -10 & 10 & 10 \end{bmatrix}$. Find each of the following. If a particular item doesn't exist, *explain why not!*

1A) The number of entries of BA.

$\underbrace{\underset{3\times4}{B}\ \underset{4\times3}{A}}_{3\times3}$ belongs to $M_{3\times3}$; has 9 entries

1B) The number of entries of $A + A + A$.

$A \in M_{4\times3}$; adding to itself is "OK" & the result is (still) in $M_{4\times3}$; this sum has 12 entries

1C) The entry in the second row, third column of AB.

is obtained from the circled row & column above as $2\cdot30 - 4\cdot20 + 7\cdot10 = 50$.

1D) The entry in the second row, third column of $A^T + B$.

note A^T is 3×4 and so B can be added to it. The 2-3 entry of A^T is 0; adding to 20 gives 20.

2. What is the inverse of $\begin{bmatrix} w & x \\ y & z \end{bmatrix}$, and what condition(s) must w, x, y and z satisfy in order for the inverse to exist?

$$\frac{1}{wz-xy}\begin{bmatrix} z & -x \\ -y & w \end{bmatrix},$$ provided $wz - xy \neq 0$.

3. Suppose A and B are both $n \times n$ matrices. Find a formula for the inverse of $(AB)^T$ in terms of A^{-1} and B^{-1}. Show all your steps.

$$((AB)^T)^{-1} = (B^TA^T)^{-1} = (A^T)^{-1}(B^T)^{-1} = (A^{-1})^T(B^{-1})^T$$

(note there is no such thing as "A^{-T}"...)

4. Suppose the inverse of A is $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 10 & 0 & 20 \end{bmatrix}$. ← so A^{-1} is this matrix...

4A) Find all solutions of $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

given that A^{-1} exists, we have

$$A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \Rightarrow A^{-1}A\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 10 & 0 & 20 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 21 \\ 26 \\ 90 \end{bmatrix}$$

4B) Do the columns of A span $\mathbf{R}^3$? Explain.

We need to know if $A\vec{x} = \vec{b}$ has a solution for ANY/EVERY $\vec{b} \in \mathbb{R}^3$. But following (4A), we have $\vec{x} = A^{-1}\vec{b}$ is indeed such a solution for any $\vec{b}$. The answer is yes.

(note $A\vec{x} = A(A^{-1}\vec{b}) = I_3\vec{b} = \vec{b}$ as desired!)