TEST 1

Math 205
10/10/11

Name: ______________________________
by writing my name i swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order if you want to get full credit (matrices can be reduced into RREF with calculator without showing steps). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements. Put a smiley face next to your name for one point.
- This test has 8 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*13 points*) Consider the system of equations:

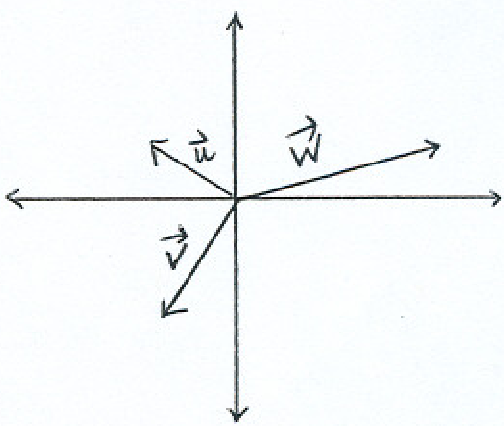
$$\begin{array}{rcrcrcrcrcr} x & + & 6y & + & 2z & - & 5u & - & 2v & = & -4 \\ & & & & 2z & - & 8u & - & v & = & 3 \\ & & & & & & & & v & = & 7 \end{array}$$

a. (*5 pts*) Write the system as a matrix equation.

b. (*8 pts*) Solve the system and use vector parameter form for your solution.

2. (*10 points*) Consider the vectors $\vec{u}, \vec{v}, \vec{w}$ as labelled on the graph.

a. (*5 pts*) Sketch and label $2\vec{u}$ and $\vec{v} + \vec{w}$.



b. (*5 pts*) Describe, geometrically and in words, the space Span$\{\vec{u}, \vec{v}, \vec{w}\}$.

3. (*20 points*)

a. (*3 pts*) Give the definition of what it means for the set of vectors $S = \{\vec{x_1}, \vec{x_2}, \ldots \vec{v_p}\}$ to be linearly independent. (Don't tell me the method you would use to SHOW they are linear independent. I am looking for the definition.)

Now, suppose $S = \{\vec{x_1}, \vec{x_2}, \vec{x_3}, \vec{x_4}\}$ be the columns vectors of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & -1 & 3 \\ 1 & -2 & 1 & 1 \\ -2 & -1 & 1 & -3 \end{bmatrix}$

b. (*5 pts*) Find all solutions of $A\vec{x} = \vec{0}$. Hint: $RREF(A) = \begin{bmatrix} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c. (*4 pts*) Express $\vec{0}$ as a linear combination of the columns of A where the scalar associated with $\vec{x}_3$ is one or explain why this is impossible.

d. (*5 pts*) Express $\vec{x}_3$ as a linear combination of the other columns or explain why this is impossible.

e. (*3 pts*) Given some vector $b \in \mathbb{R}^4$, can you always solve $A\vec{x} = \vec{b}$? Explain.

4. (*12 points*) One way to see if a linear transformation $T(\vec{x}) = A\vec{x}$ is one-to-one is to check which vectors $\vec{x}$ in the domain are mapped to the zero vector, $\vec{0}$ (ie. Find all $\vec{x}$ such that $T(\vec{x}) = A\vec{x}) = \vec{0}$.) We know $\vec{x} = \vec{0}$ will work, but there may be more in which case T is not one-to-one.

a. (*6 pts*) Find an example of a non-trivial (not the zero matrix) linear transformation that maps a non-zero vector to the zero vector. Give a matrix, A, and a vector, $\vec{x}$, and show how A maps your chosen $\vec{x}$ to the zero vector.

b. (*6 pts*) A linear transformation T is one-to-one if and only if the ONLY vector that T maps to $\vec{0}$ is $\vec{x} = \vec{0}$ (ie. only the trivial solution). Explain why if $T(\vec{x}) = 0$ has a unique solution, $\vec{x} = \vec{0}$, then T is one-to-one.

5. (*15 points*) T is a linear transformation. $T : \mathbb{R}^2 \to \mathbb{R}^3$.

a. (*8 pts*) Can T be one-to-one? Can T be onto? Explain.

b. (*7 pts*) $T(\vec{e_1}) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and $T(\begin{bmatrix} -1 \\ 1 \end{bmatrix}) = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$. Find $T(\begin{bmatrix} 3 \\ 2 \end{bmatrix})$.

6. (*10 points*) True or False. No explanation necessary.

- ____ Every elementary row operation is invertible.
- ____When $\vec{u}$ and $\vec{v}$ are nonzero vectors, Span$\{\vec{u}, \vec{v}\}$ contains the line through $\vec{u}$ and the origin.
- ____Whenever a system has free variables, the solutions set contains many solutions.
- ____The columns of a 4 x 2 matrix always span $\mathbb{R}^2$.
- ____My favorite theorem used to be Theorem 4, but now it is the Invertible Matrix Theorem.

7. (*10 points*) Given A is an $m \times n$ matrix and B is an $r \times t$ matrix.

a. (*2 pts*) Explain what must be true of m, n, r and t for AB to exist.

b. (*2 pts*) Explain what must be true of m, n, r and t for BA to exist.

c. (*4 pts*) Explain what must be true of m and n for A^2 to exist. What about for A^T to exist?

d. (*2 pts*) Suppose $T(\vec{x}) = A\vec{x}$. T maps $\mathbb{R}^? \to \mathbb{R}^*$. What is ? and what is $\star$?.

8. (*10 points*) Find all a,b,c $\in \mathbb{R}$ (a,b,c $\neq 0$) for which the matrix $A = \begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$ solves the equation $A^2 - a * A^T = I_2$ where I_2 is the 2×2 identity matrix.