1. Let $T : \mathbf{R}^a \to \mathbf{R}^b$ be the linear transformation defined as $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 2 & 2 & 1 \\ 9 & 7 & 3 \\ 4 & 0 & -1 \\ 11 & 5 & 1 \end{bmatrix}$

1A. What are a and b? $a = \boxed{}$ $b = \boxed{}$

1B. Find $T(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix})$.

1C. Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. Find any/all conditions that b_1, b_2, b_3, and b_4 must satisfy in order for $\mathbf{b}$ to be in the range of T.

1D. Verify that $\mathbf{s} = \begin{bmatrix} 0 \\ 5 \\ 10 \\ 15 \end{bmatrix}$ satisfies the condition(s) in 1C.

1E. Find all $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{s}$.

NOTE: This is problem 1 continued!

1F. Suppose $\mathbf{d} = \begin{bmatrix} -3 \\ 2 \\ d_3 \\ d_4 \end{bmatrix}$. Use the conditions in (1C) to find all values of d_3 and d_4 for which $\mathbf{d}$ is in the range of T. (Note you will be setting up a little linear system, and you should use our linear algebra techniques to solve it).

1G. Is T *onto* $\mathbf{R}^b$? Explain why or why not.

1H. Is T one-to-one? Explain why or why not.

2. Let $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_6$ be the column vectors of $Q = \begin{bmatrix} 6 & 7 & 10 & 1 & 8 & 17 \\ 4 & 6 & 12 & 2 & 5 & 18 \\ 3 & 3 & 3 & 1 & 4 & 11 \\ 2 & 1 & -2 & 0 & 3 & 4 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 79 \\ 66 \\ 36 \\ 15 \end{bmatrix}$

2A. Use your calculator to find rref($[Q|\mathbf{b}]$) and copy the result here:

2B. Find all solutions of $Q\mathbf{x} = \mathbf{b}$ and express them in the parametric form $\mathbf{x} = \mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $Q\mathbf{x} = \mathbf{b}$ and $\mathbf{v}_h$ gives all solutions of the corresponding homogeneous equation. *Circle and label* the two parts of your answer.

2C. Show that $\mathbf{c}_2$ can be expressed as a linear combination of the other five column vectors; give an explicit LC.

2D. At least one of the column vectors is not a linear combination of the others. Find such a vector and explain why it can't be so expressed.

2E. Is the span of the set of column vectors of Q all of $\mathbf{R}^4$? Explain your answer.

2F. Is the set $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_6\}$ linearly independent? Explain why or why not.

3. A model for an economy uses 4 sectors A, B, C, D. Sector B consumes 10% of the output of A, twice that much of its own output, and 2/5 of D's output. Sector C uses 1/10 of its own output and the remainder is consumed in equal portions by the other three sectors. Sector A consumes half of D's output and *vice versa*; D also uses 1/2 of B's output but A and D consume none of their own output. Sector C consumes as much of B's output as B itself does.

3A. Remembering that the entries of each column sum to one, what is the exchange table for this economy?

3B. Suppose sector D has an equilibrium price of \$179 billion. What are the other three equilibrium prices P_A, P_B and P_C? Label your answers.

4. Suppose $T : \mathbf{R}^a \to \mathbf{R}^z$ is a transformation. Give the definitions of each of the following:

4a. T is a *linear* transformation.

4b. T is onto $\mathbf{R}^z$.

4c. Suppose $T : \mathbf{R}^3 \to \mathbf{R}^4$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2x_3 + {x_1}^2 \\ 0 \\ 3x_1 + 2x_2 + x_3 \\ 2x_2 + 7 \end{bmatrix}$. Show by example that T is not a linear transformation and that it actually fails both parts of the definition in (4a); use all-different numbers as entries in any vectors you use.

5. Suppose the solutions of a matrix equation $A\mathbf{x} = \mathbf{b}$ are written in the form $\mathbf{p} + \mathbf{v}_h$, where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}_h$ gives all solutions of the corresponding homogeneous equation.

Suppose $\mathbf{b} = \begin{bmatrix} 2 \\ -13 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ 0 \\ 6 \\ -5 \\ 0 \end{bmatrix}$ and $\mathbf{v}_h = x_2 \begin{bmatrix} 8 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -7 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, where x_2 and x_5 are free.

5A. Although we don't know what the original A is, it is possible to say what the RREF of the augmented matrix $[A|\mathbf{b}]$ is. Give it:

5B. Label the columns of (the unseen) A as $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_k$. Is the set $S = \{\mathbf{c}_1,\ \mathbf{c}_2,\ \ldots,\ \mathbf{c}_k\}$ *linearly independent*? Explain in terms of the definition of linear independence.