Math 205A Test 1 (50 points)

Name: _____

- Check that you have 7 questions on three pages.

- Show all your work to receive full credit for a problem.

1. (7 points) Let A be a 7×4 matrix such that the equation $A\vec{x} = \vec{0}$ has infinitely many solutions.

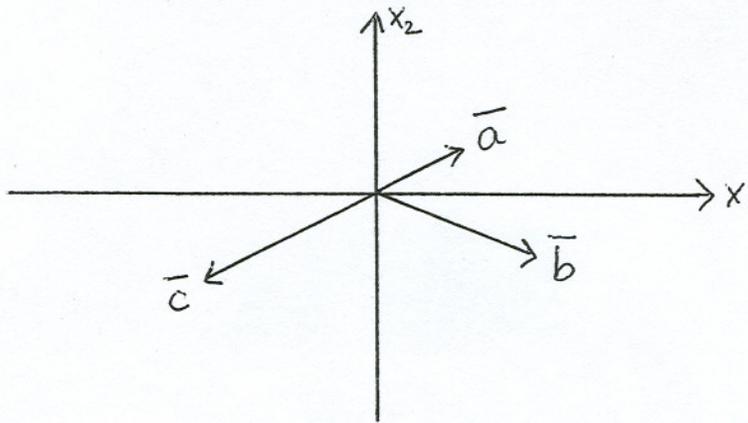   (a) Are the columns of A linearly independent? Explain.

    (b) Would it be possible to express one of the columns of the matrix A as a linear combination of the other columns? Explain.

    (c) Do the columns of A span $\mathbb{R}^7$? Explain.

2. (5 points) Suppose A, B, C and X are invertible $n \times n$ matrices and $(BX)^T - A = C$. Solve for X.

3. (5 points) Suppose the set $\{\vec{v}_1, \vec{v}_2\}$ is a linearly independent set of vectors. Is the set $\{2\vec{v}_1 - \vec{v}_2, \vec{v}_2\}$ linearly independent? Explain.

4. (8 points) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be the vectors in $\mathbb{R}^2$ shown in the figure.

(a) Give a geometric description of Span$\{\vec{a}\}$.

(b) Is $\vec{c}$ in Span$\{\vec{a}, \vec{b}\}$? Explain.

(c) Is the set $\{\vec{a}, \vec{b}, \vec{c}\}$ linearly independent? Explain.

5. (8 points) Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 1 \end{bmatrix}$. Let T be a linear transformation given by $T(\vec{x}) = A\vec{x}$

   (a) What are the domain and codomain of T?

   (b) Is T one-to-one? Explain.

   (c) Let $\vec{d} = \begin{bmatrix} -7 \\ 0 \end{bmatrix}$. Is $\vec{d}$ in the range of T? If so, how many vectors in the domain get mapped to $\vec{d}$? Explain your answers.

6. (8 points) Define a linear transformation T by the formula $T(x_1, x_2, x_3) = (x_1 + x_2 - 3x_3, 2x_1 + 2x_3, -x_2 - x_3)$.

   (a) Write the standard matrix for T.

   (b) If you were to reduce the standard matrix you wrote in part(a) by hand (and not with a calculator), what would be the first row operation to do?

   (c) Show that T is invertible and find a formula for T^{-1}.

7. (9 points) The vitamin A, vitamin C and calcium contained in one gram of four different foods is as follows.

Food 1 has 10 mg of vitamin A, 50 mg of vitamin C, and 60 mg of calcium per gram of food.

Food 2 has 30 mg of vitamin A, 30 mg of vitamin C, and 20 mg of calcium per gram of food.

Food 3 has 20 mg of vitamin A, 25 mg of vitamin C, and 40 mg of calcium per gram of food.

Food 4 has 10 mg of vitamin A, 10 mg of vitamin C, and 25 mg of calcium per gram of food.

(a) Write a system of equations to answer the question: how much of each food should a nutritionist use to prepare a meal that provides 200 mg of vitamin A, 250 mg of vitamin C, and 300 mg of calcium? State clearly what the variables represent.

(b) Write the general solution of the system you wrote in part (a) in parametric vector form. Based on the general solution, give two possible answers to the question in part (a).