1. Let \( \mathbf{a} = (2, 5, 7) \) and \( \mathbf{b} = (p, 4, 1) \); suppose \( \mathbf{a} \times \mathbf{b} \) is \( (-23, 33, -17) \). Find \( p \). Show your work.

2. Let \( \mathbf{\ell}_1(t) \) be the line parameterized by \( (2, -5, 1) + t(3, -2, 5) \).

   Let the parameteric equations for line \( \mathbf{\ell}_2 \) be \( x = 4s + 5, y = 7s - 36, \) and \( z = 2s + 20 \).

2A. It’s a fact that these two lines intersect. Find \( t \) and \( s \) for which \( \mathbf{\ell}_1(t) = \mathbf{\ell}_2(s) \). Show your work.

2B. What is the point of intersection?

2C. In the form \( (x - x_0, y - y_0, z - z_0) \cdot \mathbf{n} = 0 \), what is an equation for the plane containing these two lines?
3. Let \( \mathbf{a} = (3, -2, 5) \) and \( \mathbf{b} = (4, 7, 2) \).

3a. What is the area of the parallelogram which has vectors \( \mathbf{a} \) and \( \mathbf{b} \) as two of its sides?

3b. Find the projection vector \( \text{proj}_b(\mathbf{a}) \) of \( \mathbf{a} \) onto \( \mathbf{b} \).

3c. Find a unit vector perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

3d. To the nearest 1/10 of a degree, find in degrees the angle \( \theta \) between \( \mathbf{a} \) and \( \mathbf{b} \).

3e. Let \( P \) be the point at the tip of the vector \( \mathbf{b} \). What are the cylindrical coordinates of \( P \)?
4. Let $f$ and $g$ be differentiable vector-valued functions from $\mathbb{R}$ to $\mathbb{R}^3$.

What does theorem 1.10.3 tell us that $(f \times g)'$ is in terms of $f$, $f'$, $g$ and $g'$?

5. Let $f(t) = \langle t, t^2, t^3 \rangle$, $g(t) = \langle t^4, e^t, 0 \rangle$ and $h(t) = \cos t$.

5a. Find and simplify both $(f \times g)(t)$ and $(f \times g)'$ for this specific pair of vector-valued functions.

5b. Find explicit formulas for both $f(h(t))$ and $f(h(t))'$. 

6. Consider the sphere centered at $(3, -2, 5)$ with radius 7.

   6a. What is the equation of this sphere?

   6b. What is the equation of the plane tangent to this sphere at its “south pole”?

7. Consider an object moving along a path with its position parameterized by $\mathbf{g}(t) = (g_1(t), g_2(t))$ where

$$g_1(t) = t^4 - \frac{4}{3}t^3 - 12t^2 + 40 \quad \text{and} \quad g_2(t) = t^3 - 3t.$$

(Assume $t$ is in seconds and the units on the $xy$ plane are in feet).

   7a. Use your calculator to draw a decent, labeled sketch of this path for $t \in [-2.6, 3.2]$. Set your window to $[-30, 50] \times [-12, 25]$ and set the scales on the $x$ and $y$ axes to 10 and 10.

   7b. What are all the times that the horizontal components of the velocity are 0?

   7c. Find a parameterization for the line tangent to the path at $t = 3$.

   7d. If the object leaves the path at $t = 3$ and follows the tangent line with the velocity it has at $t = 3$, where is it at $t = 7$?
7e. *Bonus!* this is a continuation of the problem on the previous page: To the nearest 1/10, when is the object moving slowest, and where is it? Explain!

8. Find the equation of the elliptic paraboloid given here: In addition to the two points labeled, it’s tangent to the $xy$ plane. Just FYI, the “contour curves” are at heights $z = 2, 4, 6, 8, \ldots$. 