Math 105 B,C          Fall 2004        MIDTERM I

Please write all answers in the space provided on the exam. If you need more space, use the back, but INDICATE CLEARLY to the grader that there is work on the back, and label clearly. You may use scrap paper, but copy all relevant work out neatly on the exam.

1. **6 points** For each of the following functions, decide if they are continuous and/or differentiable on the given intervals. (Just say continuous or not continuous, differentiable or not differentiable, you needn’t explain.)

   a. $\frac{1}{x-2}$ on $[-1, 1]$
   b. $\frac{1}{x-2}$ on $[0, 3]$
   c. $\frac{1}{|x-2|}$ on $[-1, 1]$

2. **10 points** Consider the function

   $$f(x) = \begin{cases} \frac{x}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   a. Draw a graph of $f(x)$ for $-2 \leq x \leq 2$.

   b. To say “$f$ is continuous at 0” means . . . (write out the limit definition of continuity at a point).

   c. What is $\lim_{x \to 0} f(x)$?
d. What is $f(0)$?

e. Is $f$ continuous at 0? Explain.

3. **10 points**

a. A car starts with a small positive velocity, and gradually speeds up until the driver sees a speed trap ahead and slows down. Sketch a graph of the position of the car as a function of time.

b. Sketch a graph of the height of a particle against time, if its velocity is positive and its acceleration is negative.

4. **10 points** In each part below, be sure your sketch is consistent with the important features of the graph given.

a. Below is a graph of $f(x)$. Sketch a rough graph of $f'(x)$, either on the same set of axes, or right next to it. Label your axes with tick marks, where relevant.

\[ f(x) \]
b. Below is a graph of $g'(x)$. Sketch a rough graph of $g(x)$, either on the same set of axes, or right next to it. Label your axes with tick marks, where relevant.

\[ g'(x) \]

5. 16 points

a. For an arbitrary function $g(x)$, write out the definition of $g'(0)$ (as the limit of a difference quotient).

b. Apply this definition to the function $g(x) = 7^x$. (That is, express $g'(0)$ as the limit of a difference quotient when $g(x) = 7^x$.)

c. Estimate this limit numerically using the difference quotient. (That is, take smaller and smaller positive and negative values of $h$ in the difference quotient.)
d. Find $g'(0)$ exactly. (Hint: first find $g'(x)$ using the rule for differentiating exponential functions.)

6. 10 points If $f(x) = \frac{1}{x}$, find the equation of the line tangent to $f$ at $x = 2$.

7. 8 points The cost, $C$ (in dollars) to produce $g$ gallons of ice cream can be expressed as $C = f(g)$. Using units, explain the meaning of the following statements in terms of ice cream:
   a. $f(200) = 350$
   b. $f'(200) = 1.4$

8. 30 points Find the derivatives of the following functions:
   a. $y = x^4 + 4^x$
b. \( f(x) = \pi^4 + \pi^x + x^\pi \)

c. \( h(t) = \frac{2}{t} + \frac{1}{t^5} \)

d. \( g(w) = 6^w \cdot \sqrt[4]{w} \)

e. \( f(x) = \frac{x^2 + x^2}{6x - 7} \)