Math 206 Section A

Test 1

75 points

Name: Solutions

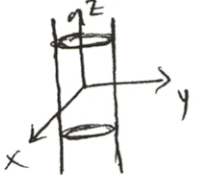
Show all your work to receive full credit for a problem.

There are eight questions. Questions are printed on both sides of a page.

1. **(10 points)** Let $x^2 + y^2 = 9$.

   (a) Describe or sketch the set of points in $\mathbb{R}^3$ that satisfy the above given equation.

   Cylinder of radius 3 with the z-axis as its axis.



   (b) Write the given equation in spherical coordinates.

   $x = \rho \sin\phi \cos\theta$ $\quad y = \rho \sin\phi \sin\theta$

   $x^2 + y^2 = 9$ becomes $\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta = 9$

   $\rho^2 \sin^2\phi = 9$.

   (c) Is the point $(0, -3, 5)$ on the surface represented by the given equation? Explain.

   $(0)^2 + (-3)^2 = 9$.

   The point satisfies the equation $x^2 + y^2 = 9$.

   So it is on the surface.

   (d) Give cylindrical coordinates of the point $(0, -3, 5)$.

   $r = \sqrt{0 + (-3)^2} = 3$

   ~~z~~ $w = z = 5$

   $\tan\theta = \frac{y}{x} = \frac{-3}{0}$, undefined.

   $\theta = \frac{3\pi}{2}$ (since the point $(0, -3)$ is on negative y-axis.)

2. **(8 points)** Describe the curve in which the two surfaces $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$ intersect. Write a parametrization for this curve.

$z = 6 - x^2 - y^2$ is an upside down paraboloid.

The two paraboloids intersect in a circle.

$z = 6 - (x^2 + y^2) = 6 - z$ gives $z = 3$.

They intersect in the plane $z = 3$.

$z = x^2 + y^2$ gives $x^2 + y^2 = 3$.

Circle has radius $\sqrt{3}$.

Parametrization for the circle:

$$x = \sqrt{3}\cos t$$
$$y = \sqrt{3}\sin t$$
$$z = 3$$