

Math 206 Section A

Test 1

75 points

Name: Solutions

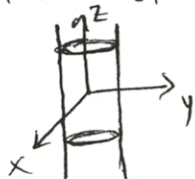
Show all your work to receive full credit for a problem.

There are eight questions. Questions are printed on both sides of a page.

1. (10 points) Let $x^2 + y^2 = 9$.

(a) Describe or sketch the set of points in \mathbb{R}^3 that satisfy the above given equation.

Cylinder of radius 3 with the z-axis as its axis.



(b) Write the given equation in spherical coordinates.

$$x = \rho \sin \phi \cos \theta \quad y = \rho \sin \phi \sin \theta$$

$$x^2 + y^2 = 9 \text{ becomes } \rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 9$$

$$\rho^2 \sin^2 \phi = 9.$$

(c) Is the point $(0, -3, 5)$ on the surface represented by the given equation? Explain.

$$(0)^2 + (-3)^2 = 9.$$

The point satisfies the equation $x^2 + y^2 = 9$.
So it is on the surface.

(d) Give cylindrical coordinates of the point $(0, -3, 5)$.

$$r = \sqrt{0 + (-3)^2} = 3$$

$$z = w = z = 5$$

$$\tan \theta = \frac{y}{x} = \frac{-3}{0}, \text{ undefined.}$$

$$\theta = \frac{3\pi}{2} \text{ (since the point } (0, -3) \text{ is on negative } y\text{-axis.)}$$

2. (8 points) Describe the curve in which the two surfaces $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$ intersect. Write a parametrization for this curve.

$z = 6 - x^2 - y^2$ is an upside down paraboloid.
The two paraboloids intersect in a circle.

$$z = 6 - (x^2 + y^2) = 6 - z \quad \text{gives } z = 3.$$

They intersect in the plane $z = 3$.

$$z = x^2 + y^2 \quad \text{gives } x^2 + y^2 = 3.$$

Circle has radius $\sqrt{3}$.

Parametrization for the circle:

$$x = \sqrt{3} \cos t$$

$$y = \sqrt{3} \sin t$$

$$z = 3.$$