

Math 206 Section A

Test 1

75 points

Name: _____

Show all your work to receive full credit for a problem.

There are eight questions. Questions are printed on both sides of a page.

1. (10 points) Let $x^2 + y^2 = 9$.

(a) Describe or sketch the set of points in \mathbb{R}^3 that satisfy the above given equation.

(b) Write the given equation in spherical coordinates.

(c) Is the point $(0, -3, 5)$ on the surface represented by the given equation? Explain.

(d) Give cylindrical coordinates of the point $(0, -3, 5)$.

2. **(8 points)** Describe the curve in which the two surfaces $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$ intersect. Write a parametrization for this curve.

3. **(10 points)** Let L be the line given by $x = 2 - t$, $y = t$, $z = t + 1$ and let P be the plane given by the equation $5x - y + 7z = 21$.

(a) Find the coordinates of two points on the line L .

(b) Find a point on the y -axis that is on the plane P .

(c) Find a normal vector to the plane P that has length 3.

(d) Does the line L intersect the plane P ? If so, where?

4. **(10 points)** Let $L_1 : x = 1 + 2t, y = 2 + 3t, z = 3 + 4t$
and $L_2 : x = 2 - t, y = 5, z = 3 - 4t$ be two lines.

(a) Find the angle between the two lines.

(b) Find the equation of the plane containing the two lines.

5. **(10 points)** Two ropes are attached to an object placed at the origin. One rope is pulled with a force of 100 newtons in a direction 60 degrees north of east. The other rope is pulled with a force of 70 newtons in the west direction. Find the total force acting on the object. Assume the positive x -axis points in the east direction and the positive y -axis points in the north direction.

6. **(10 points)** Let $v(t) = (2t^2 + 5)\vec{i} - (t + 1)\vec{j} + t^2\vec{k}$ be the velocity function of an object.

(a) Find the position function $r(t)$ that satisfies the initial condition $r(0) = \vec{i} - \vec{j} + \vec{k}$.

(b) Find a parametrization for the line tangent to the path $r(t)$ at $t = 1$. (Use the $r(t)$ that you find in part (a).)

7. (9 points) Consider the following linear transformation T on \mathbb{R}^3 :

Dilation by a factor of 2 followed by clockwise rotation about the z -axis by 30 degrees.

(a) Write a matrix for T .

(b) Is T invertible? Explain.

(c) Let P be a parallelepiped in \mathbb{R}^3 such that $\text{Volume } T(P) = 100$. Find $\text{Volume } (P)$.

8. **(8 points)** Let $p(x, y, z) = 2x^2 + 3y^2 + 5z^2 + 8xz + 6yz$ be a quadratic form on \mathbb{R}^3 .

(a) Find the symmetric matrix that represents the quadratic form.

(b) Categorize the quadratic form as positive definite, negative definite, indefinite, or none of these.