

Exam #1, Math 205B (Linear Algebra)

This take-home exam is due at class time on **Monday, October 8**. (Sooner is fine.) You may consult the textbook (or any other book) and any class notes and handouts, but **please do not discuss any details of this exam with anyone except me!** Please sign the bottom of this sheet and turn it in with your exam. You may ask me questions about the exam, but I reserve the right to give unsatisfying answers. Please show all work (though you are encouraged to *check* your answers on MATLAB or a calculator).

1. (8 points) Find the angle between the vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

2. (18 points) Solve the system $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 9 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 15 \end{pmatrix}$ by any method (by hand).

3. (20 points) Find A^{-1} (by hand) if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$.

4. (30 points) (a) Find the LU factorization of $A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 9 & 8 \\ 3 & 16 & 18 \end{pmatrix}$.

(b) Use your answer to (a) to solve $A\vec{x} = \begin{pmatrix} 7 \\ 13 \\ 15 \end{pmatrix}$.

(c) Find L^{-1} and U^{-1} . Please show work for at least one of these.

(d) Use your answers to (c) to compute A^{-1} .

5. (12 points) Let A and B be square matrices of the size, say $n \times n$ for some $n \geq 2$, and let O denote the $n \times n$ zero matrix (all of the entries of O are zero). Assume throughout this problem that $AB = O$.

(i) Show by an example that BA need not equal O . (You should be able to find a 2×2 example, though you are welcome to give a larger example instead.)

(ii) If A is invertible, must $BA = O$? Explain. What if B is invertible?

(iii) If A and B are symmetric (*i.e.*, $A = A^T$ and $B = B^T$), prove that $BA = O$.

6. (12 points) Suppose that A is a 2×2 matrix such that $A^{-1} = A^T$. Prove that A must be either a rotation matrix or a reflection matrix.

I affirm that I did not receive help from another person in doing this exam, nor did I give help to another student in the class.

(signed) _____