

MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

EXAM I - OCTOBER 7, 2015

NAME:

Section:(Circle one) A(8 : 00) B(9 : 30)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Consider the following system of linear equations

$$(1) \quad \begin{aligned} 4x_1 - 2x_2 + 7x_3 &= -5 \\ 8x_1 - 3x_2 + 10x_3 &= -3. \end{aligned}$$

(a) Find the solutions to the system (1), if it is consistent.

(b) Find the solutions to the homogeneous system

$$\begin{aligned} 4x_1 - 2x_2 + 7x_3 &= 0 \\ 8x_1 - 3x_2 + 10x_3 &= 0. \end{aligned}$$

2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation given by

$$T(x_1, x_2) = (3x_1 + 2x_2, -x_1 + 3x_2, x_1 + x_2).$$

(a) Find all \vec{x} such that $T(\vec{x}) = \vec{0}$.

(b) Determine whether T is one-to-one. Justify your answer.

(c) Determine whether T is onto. Justify your answer.

3. Let

$$B = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & 3 \end{bmatrix}.$$

(a) Are the columns of B linearly independent? Justify your answer.

(b) Do the columns of B span \mathbb{R}^2 ? Justify your answer.

(c) Write a formula for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ so that $T(\vec{x}) = B\vec{x}$ for any vector \vec{x} in \mathbb{R}^3 .

4. Use elementary row operations to find the inverse A^{-1} of the following invertible matrix

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}.$$

(Show all your steps.)

5. (a) Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a transformation given by

$$S(x, y) = (1 - xy, x + y).$$

Determine whether S is a **linear** transformation. Explain.

(b) Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -4 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}$. Find (i) AB ; (ii) BA^T ; and (iii) $A - B^T$.