TEST 1

Math 205
10/7/13

Name: ______________________
by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order if you want to get full credit (matrices can be reduced into RREF with calculator without showing steps unless otherwise indicated). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements. Put a smiley face next to your name for one point.
- This test has 7 problems and one Bonus and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*18 points*) Consider the following matrix equation.

$$\begin{pmatrix} 3 & 5 & 4 & 0 & 1 \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 8 \\ 0 \end{pmatrix}$$

a. (*5 pts*) Write the system as a vector equation.

b. (*5 pts*) Write the equation as a system of linear equations.

c. (*8 pts*) Solve the system and use vector parametric form for your solution.

2. (*10 points*) True or False. No explanation necessary.

- ____ If a system of linear equations has no free variables, then it has a unique solution.
- ____Every matrix is row equivalent to a unique matrix in row echelon form.
- ____Every matrix is row equivalent to a unique matrix in reduced row echelon form.
- ____ If $\vec{u}, \vec{v}$, and $\vec{w}$ are vectors in $\mathbb{R}^2$, then $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.
- ____ If A is an $n \times (n+1)$ matrix with n pivots, then the columns of A span $\mathbb{R}^n$.

3. (*17 points*) Determine whether the following sets of vectors are linearly independent or linearly dependent. You should be able to do this by inspection. Give a brief explanation.

a. (*3 pts*) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} \right\}.$

b. (*3 pts*) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}.$

c. (*3 pts*) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}.$

d. (*2 pts*) Do the vectors in the set in part a. span $\mathbb{R}^3$?

e. (*2 pts*) Do the vectors in the set in part b. span $\mathbb{R}^2$?

f. (*2 pts*) Do the vectors in the set in part c. span $\mathbb{R}^2$?

g. (*2 pts*) If we add the zero vector to the set in part c., does the set span $\mathbb{R}^2$?

4. (*18 points*) Let $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 17 \\ \frac{5}{2} \\ -9 \\ -2 \end{pmatrix} \right\}$. If necessary, we can call the vectors in S, $\vec{v_1}, \vec{v_2}$, and $\vec{v_3}$, respectively.

a. (*6 pts*) The vector $\vec{0}$ is always a linear combination of vectors in S because we can use the scalar 0 as the weight for every vector. Are there others (non-trivial) ways to write $\vec{0}$ as linear combination of vectors in S? If so, find all the solutions. If not, explain.

b. (*5 pts*) Express a vector in S as a linear combination of the other vectors or explain why this is impossible.

c. (*4 pts*) Given some vector $\vec{b}$ in $\mathbb{R}^4$, is every $\vec{b}$ a linear combination of the vectors in S? Explain.

d. (*3 pts*) Geometrically, what is the span of the vectors in S?

5. (*8 points*) Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and $T(\vec{x}) = A\vec{x}$.

a. (*2 pts*) The matrix A has dimensions _______ $\times$ _______.

b. (*2 pts*) What may be true of m and n if T is onto? Circle all that apply.

$m > n$ $\qquad$ $m < n$ $\qquad$ $m = n$

c. (*2 pts*) What may be true of m and n if T is one-to-one? Circle all that apply.

$m > n$ $\qquad$ $m < n$ $\qquad$ $m = n$

d. (*2 pts*) What may be true of m and n if T is both one-to-one and onto? Circle all that apply.

$m > n$ $\qquad$ $m < n$ $\qquad$ $m = n$

6. (*20 points*) Let T and U be linear transformations.

$$T(x_1, x_2) = (x_2, x_1) \text{ and } U(x_1, x_2) = (x_1, 0)$$

a. (*8 pts*) Find the matrices A_T and A_U for the transformation T and U, respectively.

b. (*4 pts*) Is T one-to-one? Is T onto? Briefly explain.

c. (*4 pts*) Is U one-to-one? Is U onto? Briefly explain.

d. (*4 pts*) Consider the transformation $(U + T)$. What is the image $\vec{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ under the transformation $(U + T)$?

7. (*8 points*) T is a linear transformation. $T : \mathbb{R}^{(2\times 2)} \to \mathbb{R}^{(2\times 2)}$. This means T maps 2×2 matrices to 2×2 matrices. Suppose

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

a. (*4 pts*) Write $\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

b. (*4 pts*) Determine $T\left(\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}\right)$.

Bonus Question (2 Extra Credit Points):

Suppose $a + b = \min(a, b)$ and $a * b = a + b$.
What is $(2 + 3) * ((-1) + (-4)) + (-4)^2$?

Scrap Page
(please do not remove this page from the test packet)