MATH 106A - CALCULUS II
FALL 2005

QUIZ 4

NAME:

Show ALL your work CAREFULLY.

(a) Use the method of Integration By Parts to evaluate the following indefinite integral.
\[ \int x^2 \ln(3x) \, dx \]

Let \( u = \ln(3x) \) and \( dv = x^2 \, dx \). Then, \( du = \frac{1}{3x} \cdot 3dx = \frac{1}{x} \, dx \) and \( v = \frac{x^3}{3} \). It follows that
\[
\int x^2 \ln(3x) \, dx \quad \text{IBP} \quad = \frac{x^3}{3} \ln(3x) - \frac{1}{3} \int x^2 \, dx \\
= \frac{x^3}{3} \ln(3x) - \frac{x^3}{9} + C.
\]

(b) Find the present value of a non-constant income stream of $1000e^{0.02t}$ per year at time \( t \) over a period of 4 years, assuming that the interest rate is 3% compounded continuously.

The income stream function is given by \( p(t) = 1000e^{0.02t} \) dollars per year. At an interest rate of 3%, the present value over a period of 4 years is given by
\[
PV = \int_0^4 p(t)e^{-0.03t} \, dt \\
= \int_0^4 1000e^{0.02t}e^{-0.03t} \, dt \\
= 1000 \int_0^4 e^{-0.01t} \, dt = 1000 \left[ e^{-0.01} \right]_0^4 \\
= 100000[1 - e^{-0.04}] \\
= $3921.056085.
\]

Date: October 7, 2005.