

Math 205B Test 1 (60 points)

Name: _____

- Check that you have 7 questions on two pages.
- Show all your work to receive full credit for a problem.

1. (10 points) (For this problem do all calculations by hand.) Determine all possible values of h and k such that the solution set of the following system

- (a) is empty (b) contains infinitely many solutions

$$2x_1 + hx_2 = 5$$

$$4x_1 + 3x_2 = k$$

2. (8 points) Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{a}_4]$ be a 3×4 matrix. Suppose $x_1 = 3, x_2 = 4, x_3 = 1, x_4 = 2$ is a solution of the equation $A\vec{x} = \vec{0}$.

(a) Are the columns of A linearly independent? Explain.

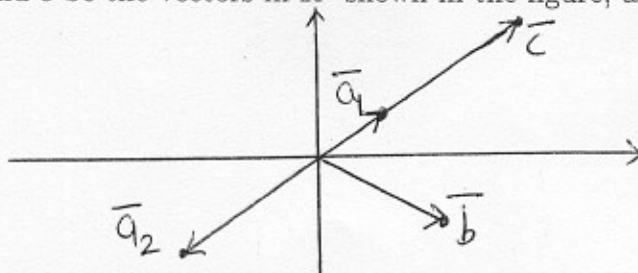
(b) Write the vector \vec{a}_4 as a linear combination of the vectors $\vec{a}_1, \vec{a}_2,$ and \vec{a}_3 .

3. (10 points) Let $A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \\ 1 & 5 & 3 & 6 \end{bmatrix}$.

(a) Describe all solutions of $A\vec{x} = \vec{0}$ in parametric vector form.

(b) Use your answer in part (a) to give two non-zero vectors that are solutions of the equation $A\vec{x} = \vec{0}$.

4. (6 points) Let \vec{a}_1 , \vec{a}_2 , \vec{b} and \vec{c} be the vectors in \mathbb{R}^2 shown in the figure, and let $A = [\vec{a}_1 \ \vec{a}_2]$.



(a) Is the vector \vec{c} in $\text{Span}\{\vec{a}_1, \vec{a}_2\}$? Explain.

(b) Does the equation $A\vec{x} = \vec{b}$ have a solution? Explain.

5. (8 points) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(x_1, x_2, x_3) = (-2x_2 + 3x_3, 5x_1 - x_3)$.

(a) Find the standard matrix of T .

(b) Is T one-to-one? Explain.

6. (6 points) Suppose a 4×4 matrix A is invertible. Explain (using pivots) why the columns of A span \mathbb{R}^4 . (If you find it helpful, you may use the following steps in your explanation.)

Since A is invertible, A reduces to _____ . So the number of pivots in A is _____

Use this to explain why the columns of A span \mathbb{R}^4 .

7. (12 points) Short answers: (No explanations needed. Simply write your answers. If you do some computation to get the answer, show the computation.)

(a) Suppose the vectors \vec{v}_1 , \vec{v}_2 , and \vec{v}_3 are in \mathbb{R}^7 . How many vectors are in $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

(b) Suppose $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ is an **onto** linear transformation. How many solutions does the equation $T(\vec{x}) = \begin{bmatrix} 80 \\ -45 \end{bmatrix}$ have?

(c) Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

- Compute $\vec{u}\vec{u}^T$.

- Compute $\vec{u}^T\vec{u}$.

(d) Find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$, if it exists.

(e) Let T be a linear transformation given by $T(\vec{x}) = A\vec{x}$, where A is a 3×5 matrix. Suppose $T(\vec{u}) = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ and $T(\vec{v}) = \begin{bmatrix} 0 \\ -5 \\ 7 \end{bmatrix}$. Find $T(4\vec{u} - \vec{v})$.