

Math 205B Test 1 (60 points)

Name: \_\_\_\_\_

- Check that you have 7 questions on two pages.
- Show all your work to receive full credit for a problem.

1. (10 points) (For this problem do all calculations by hand.) Determine all possible values of  $h$  and  $k$  such that the solution set of the following system

- (a) is empty                      (b) contains infinitely many solutions

$$2x_1 + hx_2 = 5$$

$$4x_1 + 3x_2 = k$$

2. (8 points) Let  $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{a}_4]$  be a  $3 \times 4$  matrix. Suppose  $x_1 = 3, x_2 = 4, x_3 = 1, x_4 = 2$  is a solution of the equation  $A\vec{x} = \vec{0}$ .

(a) Are the columns of  $A$  linearly independent? Explain.

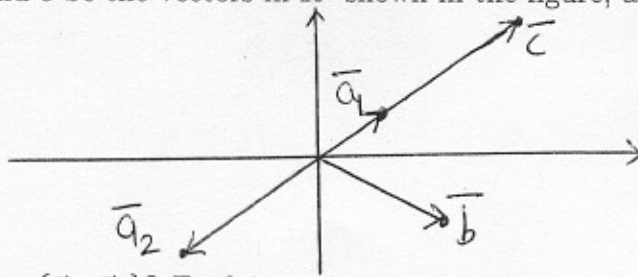
(b) Write the vector  $\vec{a}_4$  as a linear combination of the vectors  $\vec{a}_1, \vec{a}_2,$  and  $\vec{a}_3$ .

3. (10 points) Let  $A = \begin{bmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \\ 1 & 5 & 3 & 6 \end{bmatrix}$ .

(a) Describe all solutions of  $A\vec{x} = \vec{0}$  in parametric vector form.

(b) Use your answer in part (a) to give two non-zero vectors that are solutions of the equation  $A\vec{x} = \vec{0}$ .

4. (6 points) Let  $\vec{a}_1$ ,  $\vec{a}_2$ ,  $\vec{b}$  and  $\vec{c}$  be the vectors in  $\mathbb{R}^2$  shown in the figure, and let  $A = [\vec{a}_1 \ \vec{a}_2]$ .



(a) Is the vector  $\vec{c}$  in  $\text{Span}\{\vec{a}_1, \vec{a}_2\}$ ? Explain.

(b) Does the equation  $A\vec{x} = \vec{b}$  have a solution? Explain.

5. (8 points) Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation such that  $T(x_1, x_2, x_3) = (-2x_2 + 3x_3, 5x_1 - x_3)$ .

(a) Find the standard matrix of  $T$ .

(b) Is  $T$  one-to-one? Explain.

6. (6 points) Suppose a  $4 \times 4$  matrix  $A$  is invertible. Explain (using pivots) why the columns of  $A$  span  $\mathbb{R}^4$ . (If you find it helpful, you may use the following steps in your explanation.)

Since  $A$  is invertible,  $A$  reduces to \_\_\_\_\_ . So the number of pivots in  $A$  is \_\_\_\_\_

Use this to explain why the columns of  $A$  span  $\mathbb{R}^4$ .

7. (12 points) Short answers: (No explanations needed. Simply write your answers. If you do some computation to get the answer, show the computation.)

(a) Suppose the vectors  $\vec{v}_1$ ,  $\vec{v}_2$ , and  $\vec{v}_3$  are in  $\mathbb{R}^7$ . How many vectors are in  $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ ?

(b) Suppose  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  is an **onto** linear transformation. How many solutions does the equation  $T(\vec{x}) = \begin{bmatrix} 80 \\ -45 \end{bmatrix}$  have?

(c) Let  $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .

- Compute  $\vec{u}\vec{u}^T$ .

- Compute  $\vec{u}^T\vec{u}$ .

(d) Find the inverse of the matrix  $A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ , if it exists.

(e) Let  $T$  be a linear transformation given by  $T(\vec{x}) = A\vec{x}$ , where  $A$  is a  $3 \times 5$  matrix. Suppose

$T(\vec{u}) = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$  and  $T(\vec{v}) = \begin{bmatrix} 0 \\ -5 \\ 7 \end{bmatrix}$ . Find  $T(4\vec{u} - \vec{v})$ .