Exam #1, Math 205A (Linear Algebra)

This take-home exam is due by 5 PM on **Monday, October 7**. (Sooner is fine.) You may consult the textbook (or any other book) and any class notes and handouts, but **please do not discuss any details of this exam with anyone except me!** Please sign the bottom of this sheet and turn it in with your exam. You may ask me questions about the exam, but I reserve the right to give unsatisfying answers. Please show all work (though you are encouraged to *check* your answers on MATLAB or a calculator).

1. (8 points) Find the angle between the vectors $\begin{pmatrix} 1 \\ 3 \\ -5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \\ 2 \\ 2 \end{pmatrix}$.

2. (18 points) Solve the system $\begin{pmatrix} 1 & 5 & 6 \\ 3 & 4 & 1 \\ 4 & -2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 22 \\ 3 \\ 37 \end{pmatrix}$ by any method (by hand).

3. (24 points) (a) Find the LU factorization of $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 6 \\ 1 & 4 & 8 \end{pmatrix}$.

(b) Use your answer to (a) to solve $A\vec{x} = \begin{pmatrix} 5 \\ 8 \\ -12 \end{pmatrix}$.

(c) Find L^{-1} and U^{-1}. Please show work for at least one of these. (Once you've done one of them, you may be able to guess the other one.)

(d) Use your answer to (c) to find A^{-1}.

4. (24 points) (a) Find the LU factorization of the symmetric matrix $M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 6 & 4 \\ 3 & 6 & 3 & 12 \\ 4 & 4 & 12 & 4 \end{pmatrix}$.

(b) You should be able to rewrite the U from (a) as DL^T where D is a diagonal matrix and L^T is the transpose of the L in (a). What is D?

(c) Find L^{-1} and D^{-1}.

(d) Use your answers to (c) to find M^{-1}.

5. (12 points) A 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ always satisfies an equation of the form $A^2 = uA + vI$ for some scalars u and v which depend on a, b, c, d. What are u and v? Have you ever seen these functions of a, b, c, d before?

6. (8 points) If we multiply the equation $A^2 = uA + vI$ from problem 5 by A^{-1} (assuming that A^{-1} exists) it becomes $A = uI + vA^{-1}$. Plug in the u and v you got in problem 5 and then solve for A^{-1}. What do you get?

7. (6 points) What are u and v in problem 5 if A is a rotation matrix? A reflection matrix? A projection matrix? (You may make free use of your answers to project #1 here if you like.)

I affirm that I did not receive help from another person in doing this exam, nor did I give help to another student in the class.

(signed) ______________________________