

NAME:

Math 105D - Exam 1 - October 6, 2006

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (25 points)

- (a) State the limit definition of $f'(a)$.
- (b) Briefly explain your definition. What is it intended to calculate? What is the algebraic part calculating? Why is a limit necessary? You may use a graph to assist your explanation, but you must explain what the graph is indicating.
- (c) Use the limit definition of the derivative (from part a) to show that if $f(x) = x - x^2$, then $f'(3) = -5$.

2. (10 points) If $h(x) = \sqrt{3}x^7 + 7\sqrt{x} - \frac{2}{x} + 4\pi$, find $\frac{dh}{dx}$. (You do not need to use the limit definition here.)

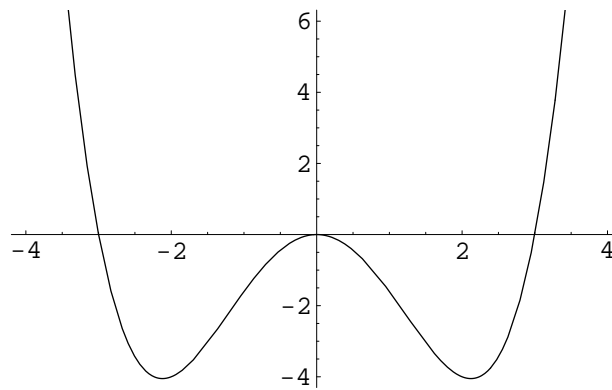
3. (10 points) Consider the second-order differential equation $y'' + \frac{y}{t} = t + 2$. Is $y(t) = t^2$ a solution? Justify your answer.

4. (16 points) Suppose that the velocity of a particle on the number line at time t is given by $v(t) = 6t^2 - 6t$. (Positive velocity means the particle is moving to the right, i.e. in the positive direction along the number line.)

(a) If the position of the particle at $t = 0$ is 5, what is the position of the particle at $t = 1$?

(b) On what interval(s) is the particle accelerating to the right?

5. (24 points) The graph of $y = f(x)$ is given below. Let $F(x)$ be an antiderivative of $f(x)$.



(a) For what value(s) of x does $f(x)$ have a stationary point?

(b) On which interval(s) is $f(x)$ increasing?

(c) On which interval(s) is $F(x)$ increasing?

(d) On which interval(s) is $f'(x)$ increasing?

(e) On which interval(s) is $F(x)$ concave up?

(f) For what value(s) of x does $F(x)$ have local maxima?

(g) Which is largest, $F(-2)$, $F(0)$, or $F(1)$?

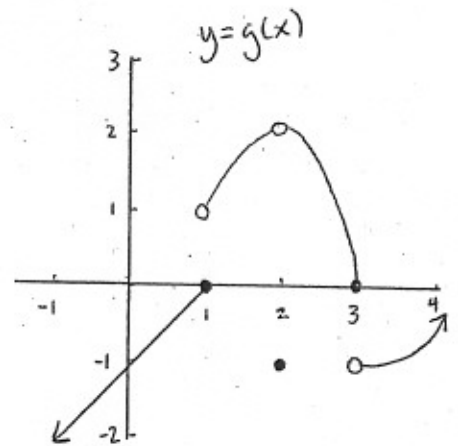
(h) Which is largest, $f'(-3)$ or $f'(-1)$?

6. (15 points) The graph of $y = g(x)$ is given to the right. For each of the following, calculate the answer or explain why it does not exist.

(a) $\lim_{x \rightarrow 1} g(x)$.

(b) $\lim_{x \rightarrow 3^+} g(x)$.

(c) $\lim_{x \rightarrow 2} (g(x) + x^2)$.



BONUS (4 points): Find a function $f(x)$ for which $f''(a) = 0$ but $x = a$ is not an inflection point of $f(x)$.

1	2	3	4	5	6	TOTAL