Math 205A Test 1 (50 points)

Name: _____

- Check that you have 6 questions on two pages.
- Show all your work to receive full credit for a problem.

1. (10 points) Let $A = \begin{bmatrix} -3 & 2 \\ 2 & -\frac{4}{3} \end{bmatrix}$.

   (a) Describe all solutions of $A\vec{x} = \vec{0}$ in parametric vector form.

   (b) Is the vector $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$ in Nul A? Explain.

   (c) Do the columns of A span $\mathbb{R}^2$? Explain.

2. (9 points) Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{a}_4]$ be a 5×4 matrix. (The vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, $\vec{a}_4$ form the columns of A.) Suppose $2\vec{a}_3 - 5\vec{a}_4 = 3\vec{a}_1 + 2\vec{a}_2$.

   (a) Is $\vec{a}_1$ in Span$\{\vec{a}_2, \vec{a}_3, \vec{a}_4\}$? Explain.

   (b) Are the columns of A linearly independent? Explain.

   (c) Find a non-zero solution of the equation $A\vec{x} = \vec{0}$.

3. (8 points) Let $H = \left\{ \begin{bmatrix} b \\ a \\ 2a - b \end{bmatrix} : a, b \text{ are real numbers.} \right\}$.

   (a) Is H a subspace of $\mathbb{R}^3$? Explain.

   (b) Give a geometric description of H.

4. (10 points) Let $A = \begin{bmatrix} 2 & 3 & -4 \\ -5 & -7.5 & 10 \end{bmatrix}$ and let T be the linear transformation given by $T(\vec{x}) = A\vec{x}$.

   (a) What is the codomain of T? Circle only one choice: $\mathbb{R}^2$ $\mathbb{R}^3$

   (b) Describe the set of all vectors $\vec{b}$ that are in the range of T. (Show all your calculations by hand.)

   (c) Is T onto? Explain.

5. (8 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2, x_3) = (2x_2 - x_3, x_1 + x_2, x_1 + 5x_3)\,.$$

(a) Find the standard matrix of T.

(b) Is T invertible? Explain. If T is invertible, find a formula for the inverse of T.

6. (5 points) Suppose A is an invertible $n \times n$ matrix such that $A^{-1} = A^T$. Then for any $n \times n$ matrix B, show that $(AB)^T A = B^T$.