Math 105: Review for Exam I

Graphical Relationships Between $f$, $f'$, and $f''$

<table>
<thead>
<tr>
<th>$f$</th>
<th>positive</th>
<th>negative</th>
<th>increasing</th>
<th>decreasing</th>
<th>concave up</th>
<th>concave down</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f''$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Let $f(x) = \frac{x + 1}{x^2 - 1}$.
   (a) What is the natural domain of $f$?
   (b) Where is $f$ discontinuous?
   (c) What is $\lim_{x \to 3} f(x)$?
   (d) What is $\lim_{x \to 1} f(x)$?
   (e) What is $\lim_{x \to 1} f(x)$?
   (f) What is $\lim_{x \to 1^+} f(x)$?

2. Let $f(x) = 3x^2 - 2x$.
   (a) Compute the average rate of change of $f$ on the interval $[2, 2.1]$.
   (b) Using the limit definition of the derivative, find $f'(x)$.
   (c) Find the equation of the tangent line to $f$ at $x = 2$.
   (d) How would the derivative of $g(x) = f(x) + 5$ compare to $f'(x)$?
   (e) How would the derivative of $h(x) = 5f(x)$ compare to $f'(x)$?
3. Given the graph of $f$, sketch a graph of $f'$ and a graph of $F$, an antiderivative of $f$ such that $F(0) = -2$.

4. Shown below is a graph of $f'$ on its entire domain. The graph is NOT $f$.

   At which $x$-value(s)
   (a) does $f$ have a stationary point?
   (b) does $f$ have a local max?
   (c) does $f$ have a local min?
   (d) does $f'$ have a stationary point?
   (e) does $f'$ have a local max?
   (f) does $f'$ have a local min?
   (g) is $f$ greatest?
   (h) is $f$ least?
   (i) is $f'$ greatest?
   (j) is $f'$ least?
   (k) is $f''$ greatest?
   (l) is $f''$ least?

   On what interval(s) is
   (a) $f$ increasing?
   (b) $f$ decreasing?
   (c) $f'$ increasing?
   (d) $f'$ decreasing?
   (e) $f$ concave up?
   (f) $f$ concave down?
5. Suppose that \( T(t) \) gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of \( T \), \( T' \), and \( T'' \) are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.

(b) The temperature is rising more and more slowly.

(c) The temperature is -5 degrees and rising.

6. An object has vertical velocity \( v(t) = t^2 - 5t + 4 \) feet per second on the interval \([0, 5]\). A positive velocity indicates the object is ascending, and a negative velocity indicates it is descending. At time \( t = 0 \), the object is 50 feet above ground.

(a) When is the object ascending? When is it descending?

(b) When is the object’s acceleration most positive?

(c) What is the greatest height the object reaches?
7. Find the derivatives of the following.
   (a) \( y = 2 + 3x + x^4 + 5x^6 \)
   
   \[ y' = 3 + 4x^3 + 30x^5 \]

   (b) \( y = \sqrt[3]{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{\pi}{6} + 6^{1/2} + \sqrt{6x^{1/6}} \)

8. Find antiderivatives of the following.
   (a) \( y = \pi + 3x^2 \)
   
   \[ \int (\pi + 3x^2) \, dx = \pi x + \frac{x^3}{3} + C \]

   (b) \( y = 4x^5 - \frac{1}{x^6} \)
   
   \[ \int (4x^5 - \frac{1}{x^6}) \, dx = x^6 + \frac{x^7}{7} - \frac{1}{5x^5} + C \]

9. Is \( y = 5x^3 \) a solution to the differential equation \( xy' - 3y = 0 \)?

10. Solve the IVP (initial value problem) \( f'(x) = -1 + x^3 \) if \( f(2) = 13 \).

See old exams and quizzes at http://abacus.bates.edu/~etowne/mathresources.html