

### Math 105: Review for Exam I

#### Graphical Relationships Between $f$ , $f'$ , and $f''$

$f$	positive	negative	increasing	decreasing	concave up	concave down
$f'$						
$f''$						

1. Let  $f(x) = \frac{x+1}{x^2-1}$ .

(a) What is the natural domain of  $f$ ?

(b) Where is  $f$  discontinuous?

(c) What is  $\lim_{x \rightarrow 3} f(x)$ ?

(d) What is  $\lim_{x \rightarrow -1} f(x)$ ?

(e) What is  $\lim_{x \rightarrow 1} f(x)$ ?

(f) What is  $\lim_{x \rightarrow 1^+} f(x)$ ?

2. Let  $f(x) = 3x^2 - 2x$ .

(a) Compute the average rate of change of  $f$  on the interval  $[2, 2.1]$ .

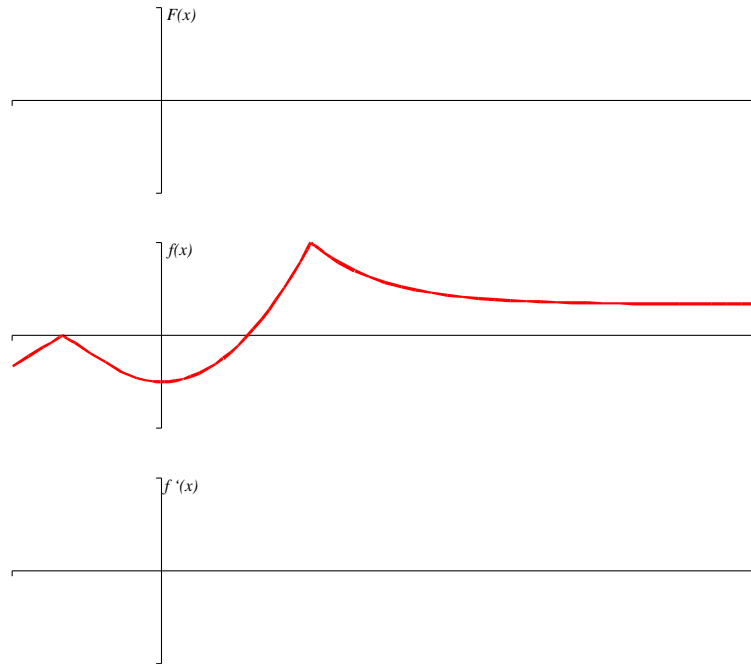
(b) Using the limit definition of the derivative, find  $f'(x)$ .

(c) Find the equation of the tangent line to  $f$  at  $x = 2$ .

(d) How would the derivative of  $g(x) = f(x) + 5$  compare to  $f'(x)$ ?

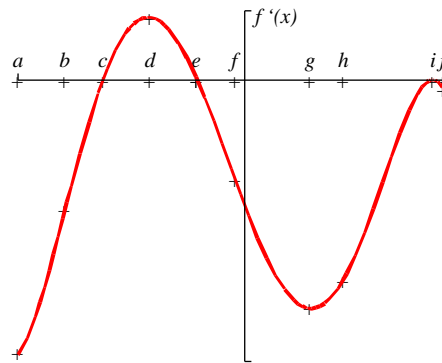
(e) How would the derivative of  $h(x) = 5f(x)$  compare to  $f'(x)$ ?

3. Given the graph of  $f$ , sketch a graph of  $f'$  and a graph of  $F$ , an antiderivative of  $f$  such that  $F(0) = -2$ .



4. Shown below is a graph of  $f'$  on its entire domain. The graph is NOT  $f$ .

- At which  $x$ -value(s)
- (a) does  $f$  have a stationary point?
  - (b) does  $f$  have a local max?
  - (c) does  $f$  have a local min?
  - (d) does  $f'$  have a stationary point?
  - (e) does  $f'$  have a local max?
  - (f) does  $f'$  have a local min?
  - (g) is  $f$  greatest?
  - (h) is  $f$  least?
  - (i) is  $f'$  greatest?
  - (j) is  $f'$  least?
  - (k) is  $f''$  greatest?
  - (l) is  $f''$  least?
- (b)  $f$  decreasing?
  - (c)  $f'$  increasing?
  - (d)  $f'$  decreasing?
  - (e)  $f$  concave up?
  - (f)  $f$  concave down?



- On what interval(s) is
- (a)  $f$  increasing?



7. Find the derivatives of the following.

(a)  $y = 2 + 3x + x^4 + 5x^6$

(b)  $y = \sqrt[6]{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{6}{x} + \frac{\pi}{6} + 6^{1/2} + \sqrt{6x^{1/6}}$

8. Find antiderivatives of the following.

(a)  $y = \pi + 3x^2$

(b)  $y = 4x^5 - \frac{1}{x^6}$

9. Is  $y = 5x^3$  a solution to the differential equation  $xy' - 3y = 0$ ?

10. Solve the IVP (initial value problem)  $f'(x) = -1 + x^3$  if  $f(2) = 13$ .