Mid-term Exam #1
MATH 205, Fall 2014

Name: ____

Instructions: Please answer as many of the following questions as possible. Show all of your work and give complete explanations when requested. Write your final answer clearly. No calculators or cell phones are allowed.

This exam has 5 problems and 100 points.

Good luck!

Problem	**Possible Points**	**Points Earned**
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	**100**	

1. (20 points) Consider the following linear system of equations:

$$\begin{cases} x_1 \quad\quad\quad\quad + 2x_3 + 4x_4 = -8 \\ \quad\quad\quad x_2 - 3x_3 - x_4 = 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 = 0 \\ \quad\quad - x_2 + 3x_3 + 4x_4 = -6 \end{cases}$$

(a) (10 points) Find the reduced echelon form of the augmented matrix associated with this system.

(b) (5 points) Write the general solution to the system in parametric vector form.

(c) (5 points) Write the general solution to the corresponding homogeneous system of equations in parametric vector form.

2. (20 points) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

(a) (10 points) Is the set $\{\mathbf{u}, \mathbf{v}\}$ linearly independent or dependent? Justify your answer.

(b) (10 points) Find all values of h such that $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$.

3. (20 points) Let $T(\mathbf{x}) : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{x_1+x_2}{2} \\ \frac{x_1+x_2}{2} \end{bmatrix}.$$

(a) (10 points) Show, using the definition of linear transformation, that T is a linear transformation.

(b) (5 points) Find A such that $T(\mathbf{x}) = A\mathbf{x}$. Show that $A^2 = A$.

(c) (5 points) Is T a mapping from $\mathbb{R}^2$ onto $\mathbb{R}^2$? *Give a one sentence explanation supporting your answer.* If T is not onto, find a vector $\mathbf{b}$ that is *not* in the range of T.

4. (20 points)

(a) (10 points) Consider the following system of equations:

$$\begin{aligned} x_1 + hx_2 &= 1 \\ 2x_1 + 3x_2 &= k. \end{aligned}$$

Choose h and k so that the system has

i. no solution;

ii. a unique solution;

iii. many solutions.

(b) (10 points) Let A be an $n \times n$ matrix. Explain why the columns of A are linearly dependent when $\det A = 0$. *Write your answer in complete sentences.*

5. (20 points)

(a) (10 points) Compute $\det(B^6)$, where $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

(b) (10 points) Show that if A is invertible, then $\det A^{-1} = \dfrac{1}{\det A}$.