Here are facts you may find useful:

For problem 1:

$$\text{The RREF of } \left[\begin{array}{rrrr|rrrrr} 3 & -11 & 1 & -3 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ -2 & 4 & -1 & 2 & 0 & 0 & 1 & 0 & 0 \\ -3 & -3 & -4 & 3 & 0 & 0 & 0 & 1 & 0 \\ 1 & -7 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad \text{is} \quad \left[\begin{array}{rrrr|rrrrr} 1 & 0 & 0 & 0 & 0 & 3 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 5 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -18 & -3 & -4 & 0 \\ 0 & 0 & 0 & 1 & 0 & -16 & -4 & -3 & -1 \\ 0 & 0 & 0 & 0 & 1 & 16 & 5 & 3 & 0 \end{array}\right]$$

For problems 2 and 3:

$$\left[\begin{array}{rrrrrcrrrr} -8 & -2 & -1 & -6 & 2 & & 1 & 0 & 0 & 0 \\ 20 & 5 & 4 & 12 & -4 & & 0 & 1 & 0 & 0 \\ -9 & -2 & -3 & -3 & 1 & & 0 & 0 & 1 & 0 \\ 24 & 6 & 9 & 6 & -2 & & 0 & 0 & 0 & 1 \end{array}\right] \text{ is row equivalent to}$$

$$\left[\begin{array}{rrrrrcrrrr} 1 & 0 & 0 & 1 & -1/3 & & 0 & 0 & -1 & -1/3 \\ 0 & 1 & 0 & 0 & 0 & & 0 & 3/7 & 4 & 8/7 \\ 0 & 0 & 1 & -2 & 2/3 & & 0 & -2/7 & 0 & 5/21 \\ 0 & 0 & 0 & 0 & 0 & & 1 & 4/7 & 0 & -1/7 \end{array}\right]$$

1. Let $B = \begin{bmatrix} 3 & -11 & 1 & -3 \\ 1 & 0 & 1 & -1 \\ -2 & 4 & -1 & 2 \\ -3 & -3 & -4 & 3 \\ 1 & -7 & 0 & -2 \end{bmatrix}$, let $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$ be the column vectors in B, and let $T : \mathbf{R}^j \to \mathbf{R}^q$ be the linear transformation which has B as its standard matrix.

1A. What are the values of j and q? $j =$ ☐ $q =$ ☐

1B. Do the columns $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$ of B form a linearly independent set? Explain your answer in terms of the definition of linear independence.

1C. Does $B\mathbf{x} = \mathbf{b}$ have a solution for every $\mathbf{b}$ in $\mathbf{R}^q$? Explain your answer.

1D. Find all solutions $\mathbf{v}_h$ of the homogeneous system $B\mathbf{x} = \mathbf{0}$.

1E. Calculate $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right)$ and call the result "**a**". $\mathbf{a} =$ ☐

1F. Find *all* solutions of $B\mathbf{x} = \mathbf{a}$ (see part 1E).

1G. Is T *onto* $\mathbf{R}^q$? Explain.

2. Let $D = \begin{bmatrix} -8 & -2 & -1 & -6 & 2 \\ 20 & 5 & 4 & 12 & -4 \\ -9 & -2 & -3 & -3 & 1 \\ 24 & 6 & 9 & 6 & -2 \end{bmatrix}$ **2A.** Find all conditions on $b_1,\ b_2,\ b_3$ and b_4 which must be satisfied in order for

$\mathbf{b} = (b_1,\ b_2,\ b_3,\ b_4)$ (written sideways to save space) to be in the span of the columns of D.

2B. Find all values of b_2 for which $\mathbf{b} = (17,\ b_2,\ 5,\ -5)$ (written sideways...) has a solution in the matrix equation $D\mathbf{x} = \mathbf{b}$.

2C. Find all values of b_3 for which $\mathbf{c} = (17,\ -31,\ b_3,\ -5)$ (sideways again) has a solution in $D\mathbf{x} = \mathbf{c}$.

2D. Verify that $\mathbf{d} = (7,\ -7,\ 8,\ 21)$ (sideways again) satisfies any/all conditions found in 2A. (Show your calculations)

2E. Write all solutions of $D\mathbf{x} = \mathbf{d}$ in the form $\mathbf{p} + \mathbf{v}_h$ where p is a particular solution of $D\mathbf{x} = \mathbf{d}$ and $\mathbf{v}_h$ is all solutions of the associated homogeneous equation.

2F. Find two more particular (or, explicit) solutions of $D\mathbf{x} = \mathbf{d}$. Show how you got them.

3. Again let $D = \begin{bmatrix} -8 & -2 & -1 & -6 & 2 \\ 20 & 5 & 4 & 12 & -4 \\ -9 & -2 & -3 & -3 & 1 \\ 24 & 6 & 9 & 6 & -2 \end{bmatrix}$. Let the column vectors of D be labeled $\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_5$

3A. Express $\mathbf{d}_2$ as a linear combination of the other columns of D or explain why this is impossible.

3B. Express $\mathbf{d}_3$ as a linear combination of the other columns of D or explain why this is impossible.

3C. Write the vector $10\mathbf{d}_1 + 20\mathbf{d}_2 + 30\mathbf{d}_3 + 40\mathbf{d}_4 + 50\mathbf{d}_5$ as a linear combination of $\mathbf{d}_1$, $\mathbf{d}_2$, $\mathbf{d}_4$ and $\mathbf{d}_5$ (that is, without using $\mathbf{d}_3$) or explain why this is impossible.

4A. Suppose that $T : \mathbf{R}^a \to \mathbf{R}^b$ is a transformation. Define what it means for T to be a *linear* transformation. Your answer will include phrases like "for all $\mathbf{v}$ in ...", etc.

4B. Suppose $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \left(\begin{bmatrix} x - 2y + 4z \\ 1 \\ 0 \end{bmatrix}\right)$. Following the approved way we did so in class, show why T is *not* a linear transformation. Does either part of the L.T. definition hold?