1. Let S be a set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ of vectors in $\mathbf{R}^m$. Define what it means to say the set S is *linearly independent.*

2. Let $A = \begin{bmatrix} 1 & 6 & 3 & 3 & 1 \\ 1 & 7 & 3 & 3 & 1 \\ 2 & 13 & 7 & 9 & 4 \\ 1 & 6 & 5 & 9 & 5 \end{bmatrix}$ and label its column vectors as $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_5$. It's a fact that the RREF of

$$\left(\left[\begin{array}{ccccc|cccc} 1 & 6 & 3 & 3 & 1 & 1 & 0 & 0 & 0 \\ 1 & 7 & 3 & 3 & 1 & 0 & 1 & 0 & 0 \\ 2 & 13 & 7 & 9 & 4 & 0 & 0 & 1 & 0 \\ 1 & 6 & 5 & 9 & 5 & 0 & 0 & 0 & 1 \end{array}\right]\right) \text{ is } \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & -6 & -5 & 0 & -23 & 17 & -10 \\ 0 & 1 & 0 & 0 & 0 & 0 & 3 & -2 & 1 \\ 0 & 0 & 1 & 3 & 2 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right].$$

2A. Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. Use the above information to decide what conditions, if any, b_1, b_2, b_3, and b_4 must satisfy in order for $A\mathbf{x} = \mathbf{b}$ to have a solution.

2B. Without doing any work, find an easy solution of $A\mathbf{x} = \mathbf{c}_3$.

2C. Since $A\mathbf{x} = \mathbf{c}_3$ does have a solution, the entries of $\mathbf{c}_3$ must satisfy the conditions in (2A). Show that this is so.

3. Let A and $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_5$ be as in problem (2).
3A. Is the set $S = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_5\}$ linearly independent? Explain.

3B. Show that it is possible to write $\mathbf{c}_3$ as a linear combination of the other four vectors. (Give an explicit LC).

3C. Explain why it is impossible to write $\mathbf{c}_2$ as a linear combination of the other four vectors.