

Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let $u =$ a function that's "inside" another function, especially if du (possibly off by a multiplying constant) is also present in the integrand.]

(a) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(b) $\int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) dx$

(c) $\int \frac{7x^2}{1+x^6} dx$

(d) $\int_6^{10} x\sqrt{10-x} dx$

2. Suppose $f(x)$ is decreasing and concave up.

(a) Put the following quantities in ascending order.

$$L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) dx$$

(b) What can you say with certainty about where S_{200} would fit into your list above?

3. Suppose $f(t)$ is the rate of change (in animals per month) of a population $P(t)$.

(a) What does $\int_4^{12} f(t) dt$ represent in this problem?

(b) Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_4^{12} f(t) dt$ given the data in the table below.

t	4	6	8	10	12
$f(t)$	15	11	8	4	3

4. Find bounds for each of the following errors if $I = \int_2^7 \ln x dx$.

(a) $|I - L_{100}|$

(b) $|I - T_{100}|$

(c) $|I - M_{100}|$

5. If $I = \int_2^7 \ln x dx$, how many subdivisions are required to obtain a trapezoidal sum approximation with error of at most $1/1,000,000$?

6. Use Euler's method with three steps on the differential equation $\frac{dy}{dt} = y - t$ to estimate $y(2.5)$ if $y(1) = 0$.

7. Solve the differential equation $dy/dx = 2xy + 6x$ if the solution passes through $(0, 5)$.

8. Write integrals equal to

(a) the arc length of $y = x^2$ on the interval $[1, 5]$

(b) the area bounded by $y = x^2 - 8x + 24$ and $y = 3x$

9. Consider the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$. Write an integral equal to the volume generated if this region is rotated about

(a) the x -axis

(b) the line $x = -1$

10. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to

(a) the volume of the pyramid

(b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (62.4 pounds per cubic foot)